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University of Strathclyde

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Welcome to Glasgow!

Dear colleagues,

It is a pleasure to welcome you to the eighth International Workshop on Lot Sizing, to the University of Strathclyde Business School, and to the friendly city of Glasgow. Hopefully, you will have “nae bother” during your stay (and in particular understanding Glaswegian!).

As in the previous editions, the workshop aims to cover recent advances in lot sizing, including new approaches for classical problems, new relevant problems, integration of lot sizing with other problems, presentation of case studies, and so on. The workshop will also aim at favoring exchanges between researchers and enhancing fruitful collaboration. Hence, we continue the tradition of the previous workshops to discuss high quality research in a relaxed atmosphere, and to allow plenty time around formal talks to maximize discussions and collaborations.

We would like to thank our generous sponsors for their support in organizing this workshop: University of Strathclyde Business School for providing us the most suitable space; and EURO, The OR Society, EURO Working Group on Lot-Sizing and ORGS for enabling free registrations to students and a number of travel grants as well as keeping costs low.

We wish you a pleasant stay in Glasgow and hope that you find the workshop inspiring and productive.

Kerem, Roberto, Mahdi and Ashwin

IWLS 2017 Program

Day 1: Wednesday, August 23, 2017

09:00-09:30 **Registration**

09:30-09:45 **Opening & Welcome**

09:45-10:45 **Session - Heuristics (chair: Ashwin Arulselvan)**

10:45-11:15 **Coffee Break**

11:15-12:45 **Session - Stochastic I (chair: Steven Prestwich)**

12:45-13:30 **Lunch**

13:30-14:30 **Session - Reformulations and Relaxations (chair: Kerem Akartunalı)**

14:30-18:00 **Distillery Tour**

Day 2: Thursday, August 24, 2017

Day 3: Friday, August 25, 2017

An easy construction heuristic for capacitated lot-sizing problems

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Abstract

The main ideas of the most successful construction heuristics, such as the Dixon-Silver-heuristic or the ABC-heuristics, is building up a production plan stepwise starting from the first period. But neither is this approach forced by the problem definition, nor is it intuitive. More naturally, one would start to plan difficult periods first, but it is unclear how a difficult period should be defined. So, our approach is to build up a production plan stepwise by adding demand elements, i.e., demands for a certain product in a certain period. Adding a single demand element to an existing production plan is straight forward. Hence, if the demand elements are added in the right order, we can create a high-quality production plan very fast.

1 Introduction

When considering fast construction heuristics for the standard capacitated lot-sizing problem (CLSP) there are two old, but still state-of-the-art algorithms. The first one is the Dixon-Silver-heuristic (cf. [1]) which builds up a production plan period-by-period starting from the first one. The main idea is to make a "local" decision about the extension of a production lot in the current period. This decision is influenced by potential per-period cost savings of the lot and the necessity of preproduction due to capacity reasons.

The second algorithm is the ABC-heuristic which consists in fact of a pool of 72 variants of a construction heuristic. These variants differ in the order of the products to be lot-sized and the lot-sizing rule. Basically the production plan is constructed step wise from the first to the last period similar to the Doxon-Silver-heuristic.

Since in the case of the CLSP with setup times, i.e. a setup operations consumes capacity, there is no easy way to determine necessary amounts to be preproduced to guarantee feasibility of the production plan[1], both algorithms are restricted to CLSP instances without setup times.

[1] For the CLSP with setup times the feasibility problem, i.e., the problem to determine if a feasible production plan exists, is NP-complete.

Recently, [3] proposed a simple construction heuristic embedded in a metaheuristic for the practical lot-sizing problem of a pharmaceutical company. This construction heuristic is a simple, rule-based method to add new demand to an existing production plan. The sequence of the demands added to the plan is optimized by a genetic algorithm. Our heuristic is based on similar ideas for the classical CLSP with setup times. We do not consider backorders but overtime in order to be able to create production plans even if there is not sufficient capacity.

2 Rule-based construction heuristic for the CLSP

The main idea of the algorithm is to add step-by-step demand to a production plan. So the first step is taking all positive demands $d_{i,t} > 0$ for all products i and all periods t and put them in a list $dl =< d_{i_1,t_1}, d_{i_2,t_2}, \ldots, d_{i_K,t_K} >$ of K elements. We start with an empty production plan, i.e., no setup and no production is scheduled yet. One-by-one a demand from the list is added to the production plan. Hence, the subproblem to solve when adding a demand to the production plan is: What is the best way to extend the current production plan such that the total cost are kept low? We apply the following rules when extending the production plan by a demand element $d_{i,t}$:

Rule 1 (use inventory) If there is positive inventory at the end of period t ($I_{i,t} > 0$) use the inventory to satisfy as much of the demand $d_{i,t}$ as possible. The amount, by which the inventory is decreased creates a new demand of the same amount for the next period and is added to the beginning of the list. If there is demand left in the current period which cannot be satisfied by the inventory add it to the production plan using the following rules.

Rule 2 (extend current lot) If there is already a lot scheduled for product i in period t extend the lot such that either the whole demand is satisfied or the capacity is used up. If there is unsatisfied demand left, increase the inventory of the previous period $I_{i,t-1}$ accordingly, and create a demand element at the front of the list. If the current period is $t = 0$ produce the whole demand using overtime.

Rule 3 (create new lot or increase previous lots) If there is no lot scheduled in period t for product i but there would be sufficient capacity for satisfying the whole demand, then create a new lot unless there is sufficient capacity in previous periods to increase already existing lots for product i and the additional holding cost are less than the setup cost.

Rule 4 (create new lot in previous period) Check if there is a period before period t, where the whole demand can be produced. If the total cost (setup cost and holding cost) is smaller than splitting the demand over periods with already existing lots then create this new lot, otherwise extend existing lots.

Rule 5 (create partial lot) Create a new lot as big as possible in period t and create a new demand for the remaining amount in period $t-1$. This demand is added to the front of the demand list.

For each element of the demand list *dl* the rules are applied one after each other starting from Rule 1 until the demand is integrated in the production plan. The final production plan depends on the order of the demand elements in the demand list. Overtime may be used in the first period only.

3 Properties and extensions

The order of the demand elements is crucial. Hence, finding a good solution for the CLSP is reduced to determine an appropriate order of the demand elements. Some preliminary tests have shown that in case of low or medium capacity utilization almost any sequence creates a reasonable good production plan without overtime. If there is a high capacity utilization it is more difficult to find a sequence which creates plan within the capacity limits.

In order to determine the optimal sequence different methods are possible. First, a simple priority rule might be sufficient to create fast reasonable good solutions. Second, any optimization method could be applied to this scheduling problem. First test with a simple local search method based on an adjacent pairwise interchange and first improvement strategy provide promising results that such a simple optimization strategy provides already high quality results.

The rules listed in Section 2 can easily be changed or extended in order to solve other lot-sizing problems. For instance, an easy way to tackle multi-level problems is to apply the above rules and with every lot of a product created in the plan, demand elements for the items on the production level below are created and added to the list. Other possibilities are to consider backorders or use the above scheme for solving an on-line lot-sizing problem where demand orders have to be scheduled immediately when they arrive.

References

[1] Dixon, P.S., Silver, E.A., A heuristic solution procedure for the multi-item, single-level, limited capacity, lot-sizing problem, Journal of Operations Management 2, 23-39 (1981)

[2] Maes, J., Van Wassenhove, L.N., A simple heuristic for the multi item single level capacitated lotsizing problem, Operations Research Letters 4, 265-273 (1986)

[3] Oyebolu, F.B., van Lidth de Jeude, J., Siganporia, C., Farid, S.S., Allmendinger, R., Branke, J., A new lot sizing and scheduling heuristic for multi-site biopharmaceutical production. Journal of Heuristics 23, 231-256 (2017)

A Unified Decomposition Matheuristic for Assembly, Production and Inventory Routing

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Abstract

We introduce a model for the assembly routing problem (ARP), which consists of simultaneously planning the assembly of a finished product at a plant and the routing of vehicles collecting materials from suppliers to meet the inventory requirements imposed by the production. We formulate the problem as a mixed-integer linear program and propose a three-phase decomposition matheuristic that relies on the iterative solution of different subproblems. The first phase determines a setup schedule while the second phase optimizes production quantities, supplier visit schedules and shipment quantities. The third phase solves a vehicle routing problem for each period. The algorithm is flexible and we show how it can also be used to solve two well-known problems related to the ARP: the production routing problem (PRP) and the inventory routing problem (IRP). Using the same parameter setting for all problems and instances, we obtain many new best known solutions out of 2,628 standard IRP and PRP test instances. In particular, on large-scale multi-vehicle instances, the new algorithm outperforms specialized state-of-the-art heuristics for these two problems.

1 Introduction

The literature on production planning has paid a lot of attention in the past decade to the integration of lot sizing and outbound transportation decisions. The typical supply chain that is considered consists of a plant that delivers final products to several customers. Considering both the production planning at the plant and the outbound delivery to the customers via routes results in what is called the production routing problem (PRP) [2]. If the production quantities at the plant are assumed to be given and the decisions only relate to the inventory and route planning, the problem is referred to as the inventory routing problem (IRP) [3].

In contrast, only few studies have focused on the integration of production planning with inbound transportation planning. Yet, in a standard supply chain, a plant often uses several different components to assemble a final product. These components are typically produced in other plants or purchased from suppliers. If the assembly plant is responsible for organizing the inbound transportation of the various components, then gains can be achieved by integrating the production planning with the inbound vehicle routing. We refer to this problem as the assembly routing problem (ARP).

To the best of our knowledge, the problem of jointly optimizing production planning and inbound vehicle routing with a finite horizon and discrete planning periods has only been considered by [4] in a restricted just-in-time (JIT) environment. They consider multiple components and products and the JIT environment assumes that the components which arrive at the plant must be used immediately in production. Therefore, [4] do not consider the possibility of holding inventory of the components at the plant level before production takes place. Furthermore, the holding cost at the suppliers is not considered in their specific study.

2 The Assembly Routing Problem

We consider a many-to-one assembly system where n suppliers each provide a unique component necessary for the production of a final product at the central plant. The planning horizon comprises a finite number of discretized time periods. The component supply at each supplier in each period is predetermined over the planning horizon. The production system has to satisfy the external demand for the final product at the plant in each period without stockouts while respecting the plant's production capacity. Both the suppliers and the plant can hold inventory. Each supplier has a storage capacity for its components. The plant provides a shared capacitated storage for the components and has a separate outbound storage capacity for the final product. A fleet of m homogeneous vehicles, each with a capacity of Q, is available to perform shipments from the suppliers to the plant using routes that start and end at the plant. We suppose throughout that the components delivered to the plant in period $t \in T$ can be used for production in the same period.

We assume that one unit of each component is needed to make one unit of the final product. Obviously, the unit components may not have identical sizes. Therefore, we consider that each component has a specific unit size which will be taken into account in the vehicle capacity and plant storage area for components. We consider a unit production cost and setup cost at the plant level. The unit holding costs are imposed for the inventory of a component at its supplier and at the plant, respectively. The inventory of the final product incurs a unit holding cost at the plant. When a vehicle travels from location i to j it entails a period-independent cost of c_{ij}.

In the ARP, the following decisions should be optimized simultaneously for each period:
1. whether or not to produce the final product at the plant and the quantity to be produced;
2. the quantity to be shipped from the suppliers to the plant, and;
3. which suppliers to visit, in what order and by which vehicle.

3 Heuristic

We present a unified decomposition matheuristic for the ARP, which can also be applied to the PRP and the IRP. In this section, we explain the algorithm in the context of the ARP.

Our algorithm decomposes the ARP model into three separate subproblems. The first subproblem is a special lot-sizing problem that determines a setup schedule by relying on an approximation of the routing cost using the number of dispatched vehicles. Considering a given setup schedule, the second subproblem chooses the node visits and shipment quantities. For multi-vehicle instances, a modified model is employed in this phase to look for possible improvements in node visits and shipments. Finally, the third subproblem solves a series of separate vehicle routing problems, one for each period t (VRP_t).

The solutions of the routing subproblems are then used to update the transportation cost approximation. This procedure is repeated for a number of iterations to reach a local optimum. Then, a local branching scheme is used to change the setup schedule and explore other parts of the feasible solution space, looking for better solutions. The entire procedure continues until a stopping condition is met.

Our algorithm shares similarities with the decomposition-based heuristic developed by [1] for the PRP. However, there are also some important differences between the two algorithms, such as the decomposition in three phases and the use of the diversification constraints.

We test our algorithm on three different problems, the IRP, the PRP and the

ARP, with a total of 4,068 instances. Updated results will be presented during the workshop.

References

[1] N. Absi, C. Archetti, S. Dauzère-Pérès, and D. Feillet, A two-phase iterative heuristic approach for the production routing problem, Transportation Science, 49 (4), 784-795 (2014)

[2] Y. Adulyasak, J.-F. Cordeau and R. Jans, Formulations and branch-and-cut algorithms for multivehicle production and inventory routing problems, INFORMS Journal on Computing, 26, 103-120 (2014)

[3] C. Archetti, N. Boland, and M. G. Speranza, A matheuristic for the multi-vehicle inventory routing problem, INFORMS Journal on Computing, 29 (3), 377-387 (2017)

[4] F. Hein and C. Almeder, Quantitative insights into the integrated supply vehicle routing and production planning problem, International Journal of Production Economics, 177, 66-76 (2016)

A stochastic multi-item lot-sizing problem with bounded number of setups

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Abstract

Within a partnership with a consulting company, we address a production problem modelled as a stochastic multi-item lot-sizing problem with bounded numbers of setups per period and without setup cost. While this formulation seems to be rather non-standard in the lot-sizing landscape, it is motivated by concrete missions of the company. Since the deterministic version of the problem is $\mathcal{NP}$-hard and its full stochastic version clearly intractable, we turn to approximate methods and propose a repeated two-stage stochastic programming approach to solve it. Using simulations on real-world instances, we show that our method gives better results than current heuristics used in industry. Moreover, our method provides lower bounds proving the quality of the approach. Since the computational time are small and the method easy to use, our contribution constitutes a promising response to the original industrial problem.

1 Introduction

Fixing the production level for the forthcoming week is a basic decision to be taken when managing a production line. Usually, a demand has to be satisfied at due dates but the limited capacity of the line prevents to produce at the last moments. On the other hand, too early productions may lead to unnecessary high inventory costs. The challenge of this kind of *lot-sizing problems* consists in finding a trade-off between demand satisfaction and holding costs. This is a well-studied topic, with many variations (deterministic/stochastic, single/multi item, etc.). Recent surveys have been proposed by Gicquel in her PhD thesis and Mula et al. (*International Journal of Production Economics*, 2006). When several references can be produced on a same line (*multi-item* lot-sizing problems), the capacity is often all the more reduced as the number of distinct references produced over the current week is high.

This is because changing a reference in production stops the line for a moment. This additional capacity reduction is usually modelled by setup costs contributing to the total cost.

The present work introduces a stochastic multi-item lot-sizing problem met within a partnership with a consulting company. A non-standard feature of the problem is that the capacity reduction due to reference changes is not modelled by setup costs but instead by an explicit upper bound on the total number of references that can be produced over a week. According to the company, many clients aim at minimizing their inventory costs while keeping the number of distinct references produced over each week below some threshold. This is essentially because setup costs are hard to quantify and a maximal number of possible changes per week is easy to estimate. To the best of authors' knowledge, the problem addressed in the present work is original and such a bound on the number of distinct references produced over a week has not been considered by academics yet, with the notable exception of Rubaszewski et al. (*IESM*, 2011) but, contrary to our problem, their bound is an overall bound for the whole horizon and they still consider setup costs.

We propose for this problem a method that can be easily used and maintained in practice. The efficiency of the method is proved via extensive numerical experiments on real industrial data.

2 Problem formulation and model

The assembly line produces a set $\mathcal{R}$ of references over T weeks. The number of distinct references produced over a week cannot exceed N. There is also an upper bound on the total week production (summed over all references). We normalize all quantities so that this upper bound is equal to 1. The demand of reference r over week t is a random variable $\boldsymbol{d}_t^r$ (of known distribution), whose realization is revealed at the end of week t. Production of a reference r that is not used to satisfy the demand can be stored and incurs a unit holding cost h^r per week. A demand that is not satisfied by the production of the present week or by inventory can be satisfied later: backorder for any reference r is allowed and incurs a unit backorder cost γ^r per week. Note that there is no setup cost, as discussed in the introduction. For each reference r, there is an initial inventory $s_0^r \in \mathbb{R}_+$. At the beginning of each week, before the demand of each reference is revealed, the production of the week has to be fixed. The objective is to minimize in expectation the total cost over the whole horizon of T weeks.

The production problem at the beginning of week t can be modelled as (S), where the variable $\tilde{\boldsymbol{s}}_{t'}^r$ (resp. $\boldsymbol{b}_{t'}^r$) models the inventory (resp. the backorder) of reference r at the end of week t', the variable $\boldsymbol{q}_{t'}^r$ models the quantity of reference r produced over week t', and the variable $\boldsymbol{x}_{t'}^r$ takes the value 1 if the reference r is produced over week t' and 0 otherwise. The last constraint of the program, written as a *measurability* *constraint*, means that the values of the variables $\boldsymbol{q}_{t'}^r$ can only depend on the values taken before time t'. A feasible solution $(\boldsymbol{q}_{t'}^r)_{t'=t,\dots,T,\, r\in\mathcal{R}}$ of (S) provides a deterministic production $(\boldsymbol{q}_t^r)_{r\in\mathcal{R}}$ for the current week t.

$$
\begin{aligned}
\min\quad & \mathbb{E}\left[\sum_{t'=t}^{T}\sum_{r\in\mathcal{R}}\left(h^r\tilde{\boldsymbol{s}}_{t'}^r+\gamma^r\boldsymbol{b}_{t'}^r\right)\right] & \\
& \boldsymbol{s}_{t'}^r=\tilde{\boldsymbol{s}}_{t'}^r-\boldsymbol{b}_{t'}^r & t'=t,\dots,T,\ \forall r\in\mathcal{R}\\
& \boldsymbol{s}_{t'}^r=\boldsymbol{s}_{t'-1}^r+\boldsymbol{q}_{t'}^r-\boldsymbol{d}_{t'}^r & t'=t,\dots,T,\ \forall r\in\mathcal{R}\\
& \sum_{r\in\mathcal{R}}\boldsymbol{q}_{t'}^r\le 1 & t'=t,\dots,T\\
& \boldsymbol{q}_{t'}^r\le\boldsymbol{x}_{t'}^r & t'=t,\dots,T,\ \forall r\in\mathcal{R}\\
& \sum_{r\in\mathcal{R}}\boldsymbol{x}_{t'}^r\le N & t'=t,\dots,T\\
& \boldsymbol{x}_{t'}^r\in\{0,1\},\ \boldsymbol{q}_{t'}^r,\tilde{\boldsymbol{s}}_{t'}^r,\boldsymbol{b}_{t'}^r\ge 0 & t'=t,\dots,T,\ \forall r\in\mathcal{R}\\
& \sigma\left(\boldsymbol{q}_{t'}^r\right)\subset\sigma\left(\left(\boldsymbol{d}_t^{r'},\dots,\boldsymbol{d}_{t'-1}^{r'}\right)_{r'\in\mathcal{R}}\right) & t'=t,\dots,T,\ \forall r\in\mathcal{R}.
\end{aligned}
\tag{S}
$$

The deterministic version of (S) is easily shown to be $\mathcal{NP}$-hard for any fixed $N\ge 3$. (Curiously, its complexity status when $N\in\{1,2\}$ is open.)

3 Method

We propose a two-stage approximation consisting in relaxing the measurability constraint: the production decisions for the current week t can still not depend on the future, but now the subsequent production decisions depend on the future demand. This relaxation is then solved by a classical *sample average approximation*. We build a set Ω of a predetermined number m of scenarios sampled uniformly at random. Each of these scenarios is a possible realization of $(\boldsymbol{d}_t^r,\boldsymbol{d}_{t+1}^r,\dots,\boldsymbol{d}_T^r)$ for each r. We get then a mixed integer program (P), solved by any standard MIP solver, with x_t^r, q_t^r, $x_{t'}^r(\omega)$, $q_{t'}^r(\omega)$, $\tilde{s}_{t'}^r(\omega)$, and $b_{t'}^r(\omega)$ for all $t'=t,\dots,T$, $r\in\mathcal{R}$, and $\omega\in\Omega$ as variables. At week t, the production is then set to be the solution $(q_t^r)_{r\in\mathcal{R}}$ found by the solver.

4 Numerical experiments

Gurobi 6.5.1 was used to solve the MIP and the computer is a PC with CPU @ 3.40GHz and 8Go RAM.

Instances. The instances used are realistic and have been provided by a client of the partner. A horizon T of 13 weeks has been used (typical one). The demand is obtained via a *generalized autoregressive process* based on historical data $(\overline{d}_t^r)_{t\in[T],r\in\mathcal{R}}$.

The initial inventory has been set to $s_0^r = \frac{1}{3}(\overline{d}_1^r + \overline{d}_2^r + \overline{d}_3^r)$. The other parameters are provided in Table 1. The parameter C is the capacity of the line before normalization: we have thus $\gamma^r = \gamma/C$.

Three other heuristics. The first heuristic is the deterministic version of (S), where the random demand is replaced by its expectation. The second one, *cover-size*, consists in determining at $t = 0$ a value T_r for each reference $r \in \mathcal{R}$ (via an "aggregated" convex program, such as the one given by Ziegler (*Operations Research Letters*, 1982). At time t, if the inventory of reference r is below a fixed safety level, the quantity q_t^r is chosen so that the inventory of reference r exceeds the safety level of the expected demand for the next T_r weeks. Easy anticipations are allowed and in case of capacity issues, the production is postponed (backorder). The third heuristic, *lot-size*, is almost the same but with a fixed quantity ℓ_r above safety level when production is activated.

Results. The results are provided in Table 1. All quantities are in M€ and given with a confidence interval at 95% (many runs have been performed to get these intervals). The column LB provides the lower bound obtained by the optimal value at time $t = 1$ of program (P) (with $m = 1000$ and a time limit of 30 minutes for the solver). The column 2SS corresponds to the proposed method (with $m = 20$ and a time limit of 90 seconds for the solver).

Comments. Our method clearly outperforms the three other heuristics. It dramatically reduces the costs, while remaining a quite simple method.

By playing with the values of γ, our method can also be used as an approach for keeping the holding cost reasonable, while trying to ensure a good *fill rate service shortage*, measured as the sum of all backorders over time. Experiments have shown that not only on the considered instances our method ensures low holding costs, but it is also competitive regarding the fill rate service shortage. Roughly speaking, it beats lot-size and is above the cover-size performances by a small percentage.

Instances	Instance characteristics					
	$\lvert\mathcal{R}\rvert$	$\max(\overline{d}_t^r)$	C	N	h_t^r	γ
L2_1000	21	4992	10562	7	35–61	1000
L2_300						300
L2_80						80
L6_1000	22	8640	13299	8	16–23	1000

Instances	LB	2SS	Det.	Cover-size	Lot-size
L2_1000	?	4.96 ± 0.06	8.20 ± 0.13	9.17 ± 0.08	18.20 ± 0.57
L2_300	?	3.39 ± 0.03	3.82 ± 0.03	8.39 ± 0.05	10.89 ± 0.16
L2_80	?	1.98 ± 0.02	2.02 ± 0.02	8.15 ± 0.50	8.59 ± 0.04
L6_1000	54.84 ± 0.44	74.22 ± 2.79	82.95 ± 2.79	149.69 ± 3.33	117.22 ± 3.15

Table 1: Instance characteristics and results

A responsive planning approach for robust stochastic capacitated lot sizing

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Abstract

This work introduces a responsive planning approach within a rolling horizon environment for stochastic capacitated dynamic lot sizing with uncertain demand under service constraints. In the first step, a set-up pattern is determined based on the distribution parameters of demand and fixed only for a small number of periods in order to achieve a high robustness of the plan. To ensure a high delivery reliability despite uncertain demand and low holding costs at the same time, the approach calculates optimal implicit dynamic safety stocks. In the second step, actual production quantities are determined period-wise, factoring in previously realized demand. By incorporating already observed realizations of the random variables for further decisions, this approach leads to both more robust production plans and less total costs in real world stochastic production environments.

1 Introduction

Real world production environments have to deal with many uncertainties, like uncertainty regarding the forecast demand. Many lot sizing approaches, however, leave aside this stochasticity of those input parameters. Stochastic lot sizing approaches, like Helber et al. (2013)[1] and Tempelmeier and Hilger (2015)[2], however, incorporate that stochasticity. Many of them are static, as they decide on both production times and production quantities before any demand is realized. Therefore, those decisions are often premature: If the demand is underestimated, the delivery reliability will be low, while an overestimated demand would lead to high inventory holding costs. A more promising way of stochastic lot sizing is introduced by this work, as it takes prior demand realizations into account when deciding on the production quantities for subsequent periods.

Proposed is a responsive planning approach with rolling horizons for robust stochastic capacitated multi-product lot sizing with service constraints. It is designed to focus on real production environments, where decisions on products to produce in a given period have to be made several periods in advance. In the first step, the decisions on the production times are made for a small number of periods, in order to get a robust set-up pattern. The actual production quantities, however, are determined within the second step period-wise in the respective period after the demand has been realized. Unlike static approaches, the responsive approach ensures that a given service level is met, and first results show that in many problem instances it also leads to lower total costs.

2 A responsive algorithm for stochastic lot sizing

The proposed approach aims to make decisions in a way that a set-up pattern can be found, which is robust to the greatest possible extent, while there is still flexibility regarding the actual production quantities. Therefore, the model is solved within a rolling horizon environment with re-optimizations executed in every period, taking into account the demand realized so far. The following three sub-horizons of the whole planning horizon are defined:

1. **Periods with fixed production quantities:**
   Only for the period currently considered (and all prior periods), the production quantities are fixed. This ensures that all possible information on past demand realizations is regarded and a profound decision is made.

2. **Periods with fixed set-up patterns:**
   The set-up pattern is fixed only for a small number of periods in advance. This leads to high robustness, while flexibility regarding subsequent periods leads to set-up patterns adapted to prior demand realizations.

3. **Periods comprised by the optimization:**
   Due to high uncertainty regarding the remote future, it is not necessary to comprise the whole planning horizon in every optimization. On the other hand, comprising only the periods for which the set-up pattern will be fixed, would lead to myopic plans. To avoid this myopia, a certain amount of additional periods in the future is taken into account in every optimization. However, the set-up pattern as well as the production quantities for these additional periods are, at this moment, still alterable and subject to recalculations in the following periods.

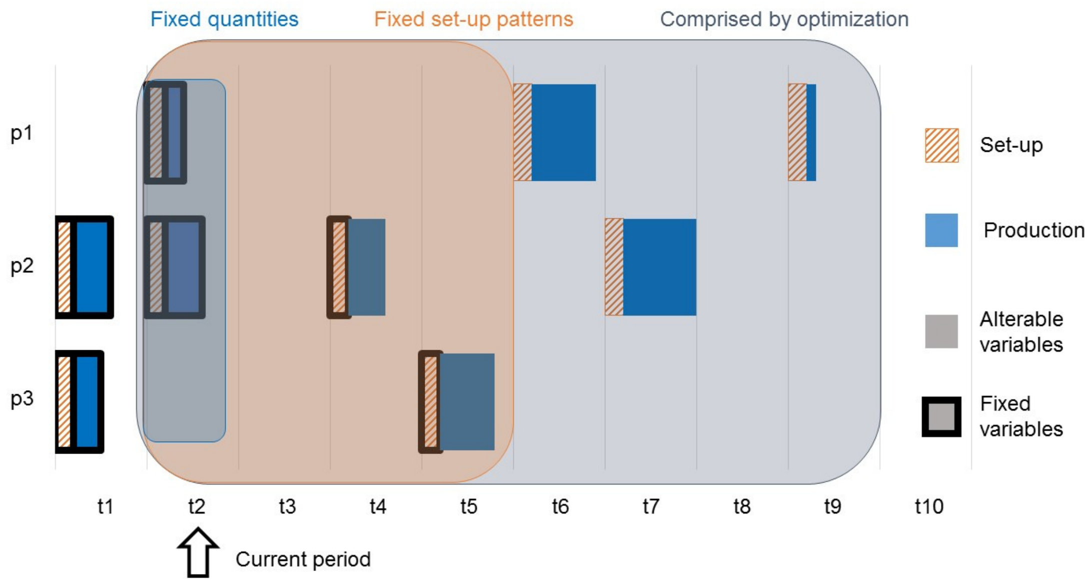

Figure 1: Exemplary plan determined by the introduced responsive algorithm

Figure 1 shows an example of a production plan determined by the introduced responsive algorithm. It depicts the results of the optimization executed in period 2. In this example it is assumed that the set-up pattern is specified four periods in advance. In order to avoid myopic production plans, the algorithm comprises four additional periods for any optimizations. So in this example in period 2 the production plan is calculated for the periods 2 to 9. However, only the set-up patterns for the four periods 2 to 5 are fixed and the plan for the additional periods 6 to 9 is still alterable. Although calculated for all the periods comprised, the production quantities are only fixed for the current period 2. This ensures that as much information about demand realizations as possible is considered when deciding on the quantities. In the following period 3 all the planning horizons still have the same lengths, but begin in period 3: The optimization then comprises periods 3 to 10 and the set-up pattern for period 6 as well as the quantities for period 3 are fixed additionally. The algorithm continues until all decisions are fixed.

In case the demand is underestimated, it might be unavoidable to produce a certain product in a given period in order to avoid a violation of the set service level, even though this is not scheduled in the fixed pattern. For this particular case, the algorithm is designed to allow an additional set-up for the respective product. These additional set-ups lead to both higher set-up costs and less robustness of the plans. Therefore, minimizing those additional set-ups is a special focus of this work.

3 Main features of the underlying model

The introduced algorithm solves an extended and generalized stochastic capacitated lot sizing model based on the SCLSP as proposed by Helber et al. (2013)[1]. The main features of the extensions include:

- **Consideration of expected costs for additional set-ups:**
  The objective function not only considers the costs for inventory holding, set-up activities and overtime, but also takes into account the expected costs for additional set-ups as a function of the production quantities and the demand information. This leads to dynamic implicit safety stocks, whose levels are chosen endogenously and depending on the current utilization. Those safety stocks reduce the total costs and increase the robustness of the production plans at the same time and therefore enhance the performance of the algorithm considerably.

- **Limitation of backlogs with waiting time constraints:**
  The known service level definitions are not applicable for the limitation of backlogs in an algorithm with varying lengths of the planning horizon. Therefore, the backlogs are controlled by a product-specific waiting time constraint. This restriction defines the maximum permitted expected mean waiting time of a a unit of demand to be fulfilled, which equals the quotient of the cumulated backlog for the respective product and of the cumulated expected demand. It leads to more consistent production and therefore to less additional set-ups and higher robustness of the determined plans.

- **Piecewise linear approximation:**
  The conceptional model for the SCLSP is non-linear and not feasible due to the random variables. Various approaches can be used to transfer the conceptional model into a linear deterministic approximation model. In this work, the piecewise linear approximation of the inventory and backlog functions as proposed by Helber et al. (2013)([1]) is adapted to varying lengths of the planning horizons.

References

[1] S. Helber, F. Sahling and K. Schimmelpfeng, Dynamic capacitated lot sizing with random demand and dynamic safety stocks, OR Spectrum, 35, 75-105 (2013)

[2] H. Tempelmeier, T. Hilger, Linear programming models for a stochastic dynamic capacitated lot sizing problem, Computers & Operations Research, 59, 119-125 (2015)

MILP-based model for approximating non-stationary (R, S) policies with correlated demands

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Abstract

This paper addresses the single-item single-stock location stochastic lot-sizing problem under (R, S) policy. We assume demands in different periods are dependent. We present a mixed integer linear programming (MILP) model for computing optimal (R, S) policy parameters, which is built upon the conditional distribution. Our model can be extended to cover time-series-based demand processes as well. Our computational experiments demonstrate the effectiveness and versatility of this model.

1 Introduction

Since [14] proved the optimality of (s, S) policies for a class of dynamic inventory models, a sizeable literature has been performed for computing the optimal policy parameters (see, for example, [20, 1, 6]). However, as pointed out in [19], although the (s, S) policy is cost-optimal, it performs poorly in terms of "nervousness", i.e. lack of planning stability. In this regard, the (R, S) policy provides an effective means of dampening the planning instability and coping with demand uncertainty. Under this policy, both inventory reviews R and associated order-up-to-levels S are fixed at the beginning of the planning horizon, while actual order quantities are decided upon only after demand has been observed.

In the seminal work, [3] proposed a two-stage deterministic equivalent heuristic which fixes replenishment periods first and then determines order quantities under the independent demand assumption. [17] presented a mixed integer programming (MIP) model that determines both timing and quantity of orders simultaneously without addressing computational performance. Under the independent demand assumption, [19, 12, 16, 17, 18, 11] proposed efficient solution methods.

In the literature, most inventory models assume that demands over different time periods are independent and identically distributed. Recently, a few studies on inventory theory with correlated demands have been emerged. They either focused on (s, S) policy (see [9, 15, 5, 4]) or measured the performance of the inventory system with specific demand patterns ([9, 7, 10, 8]). However, none of them studied the (R, S) policy with correlated demands, which motivates our work in developing an efficient method for computing (R, S) policies.

In this paper, we present an MILP-based model for approximating the (R, S) policies with correlated demand. Our model can cover a series of time-based demand process, such as the autoregressive process (AR), the moving-average process (MA), the autoregressive moving average process (ARMA), the autoregressive conditional heteroskedasticity process (ARCH). Preliminary computational experiments demonstrate that optimality gaps of our model are tighter than existing algorithms, and computational times of model are reasonable. Our model can be accommodated to approximate (s, S), and (R, Q) policies.

2 Problem description

Let a random vector $\mathbf{d} = [\mathbf{d_1}, \ldots, \mathbf{d_n}]^T$ represents stochastic demand over the planning horizon, which follows the multivariate distribution f with cumulative distribution function $F : \mathcal{R}^n \to \mathcal{R}$. Let $\tilde{\mathbf{d}}$ be the mean of demand vector $\mathbf{d}$, and $\mathbf{\Sigma}$ be the variance-covariance matrix, we require that $\mathbf{\Sigma}$ is symmetric positive definite.

Lemma 1 (Conditional distribution) *Let* $\mathbf{d} == [\mathbf{d_1}, \ldots, \mathbf{d_q}, \mathbf{d_{q+1}} \ldots, \mathbf{d_n}]^T$ *denote a random vector with joint probability function* $f(d_1, \ldots, d_q, \ldots, d_n)$*, then the conditional joint probability density function of* $d_1, \ldots, d_q$ *given* $\mathbf{d_{q+1}} = d_{q+1}, \ldots, \mathbf{d_n} = d_n$ *is*

$$f_{1,\ldots,q|q+1,\ldots,n}(d_1, \ldots, d_q | \mathbf{d_{q+1}} = d_{q+1}, \ldots, \mathbf{d_n} = d_n) = \frac{f(d_1, \ldots, d_q)}{f(d_{q+1}, \ldots, d_n)} \qquad (1)$$

We now consider the multivariate normal distribution (MVN). A vector-valued random variable $\mathbf{d} = [\mathbf{d_1}, \ldots, \mathbf{d_n}]^T$ is said to have a multivariate normal distribution (MVN) with mean $\tilde{\mathbf{d}} \in \mathcal{R}^n$ and covariance matrix $\mathbf{\Sigma} \in \mathcal{R}^{n \times n}$, if its probability density function is given by

$$f(d; \tilde{d}, \Sigma) = \frac{1}{(2\pi)^{\frac{1}{2}} |\Sigma|^{\frac{1}{2}}} \exp\big(-\frac{1}{2} (d - \tilde{d})^T \Sigma^{-1} (d - \tilde{d})^T \big). \qquad (2)$$

Lemma 2 (Conditional distribution of MVN) *Let* $\mathbf{d} = [\mathbf{d_1}, \mathbf{d_2}]^T$ *be a partitioned multivariate normal random vector, with mean* $\tilde{\mathbf{d}} = [\tilde{\mathbf{d_1}}, \tilde{\mathbf{d_2}}]^T$ *and variance-covariance matrix*

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma_{11}} & \mathbf{\Sigma_{12}} \\ \mathbf{\Sigma_{21}} & \mathbf{\Sigma_{22}} \end{bmatrix}. \tag{3}$$

Then, the conditional distribution of $\mathbf{d_2}$ *given* $\mathbf{d_1} = d_1$ *is MVN, with conditional distribution* $\mathbf{d_2}|\mathbf{d_1} = d_1 \sim \mathcal{N}\big(\tilde{\mathbf{d_2}} + \mathbf{\Sigma_{21}}\mathbf{\Sigma_{11}^{-1}}(d_1 - \tilde{\mathbf{d_1}}), \mathbf{\Sigma_{22}} - \mathbf{\Sigma_{21}}\mathbf{\Sigma_{11}^{-1}}\mathbf{\Sigma_{12}}\big)$.

Example. We now demonstrate the concepts introduced on a 4-period example. $\tilde{\mathbf{d}} = [20, 40, 60, 40]$ and standard deviations $\sigma = 0.25\tilde{\mathbf{d}}$. We assume any $\mathbf{d_t}$, $t = \{2, \ldots, T\}$, is only correlated to $\mathbf{d_{t-1}}$ with correlation coefficient $\rho = 0.5$, then the variance-covariance matrix is

$$\mathbf{\Sigma} = \begin{bmatrix} 25 & 25 & 0 & 0 \\ 25 & 100 & 75 & 0 \\ 0 & 75 & 225 & 75 \\ 0 & 0 & 75 & 100 \end{bmatrix}.$$

Therefore, the conditional distribution of $\mathbf{d_t}$, for $t = \{2, \ldots, T\}$, is $\mathbf{d_2}|\mathbf{d_1} = d_1 \sim \mathcal{N}(20 + d_1, 75)$, $\mathbf{d_3}|\mathbf{d_2} = d_2 \sim \mathcal{N}(30 + \frac{3}{4}d_2, 168.75)$, and $\mathbf{d_4}|\mathbf{d_3} = d_3 \sim \mathcal{N}(20 + \frac{1}{3}d_3, 75)$.

3 Stochastic dynamic programming

We consider a single-item single-stocking location inventory management system over a T-period planning horizon. We assume that the demand d_t depends on realised information set i_{t-1} at period $t-1$; it follows the conditional distribution $f(d_t|i_{t-1})$. Let I_{t-1} denote the opening inventory level, and Q_t represent the order quantity.

At the beginning of period t, there exist ordering costs $c(\cdot)$ comprising a fixed ordering cost K, and a linear ordering cost c. At the end of period t, a linear holding cost h is charged on every unit carried from one period to the next; a linear penalty cost b is occurred for each unmet demand at the end of each time period. Then the immediate cost can be expressed as

$$f_t(i, I_{t-1}, Q_t) = c_t(i, Q_t) + \mathrm{E}[h \cdot \max(I_{t-1} + Q_t - d_t, 0) + b \cdot \max(d_t - x - Q_t, 0)|i_{t-1} = i], \tag{4}$$

where $c_t(i, Q_t)$ is defined as:

$$c_t(i, Q_t) = K \cdot \delta_t + c \cdot Q_t, \qquad \delta_t = \{0, 1\}. \tag{5}$$

Let $C_t(i, I_{t-1})$ denote the expected total cost of an optimal policy over period $t, \ldots, T$ when the observed demand information set is $i_t = i$ and the opening inventory level is I_{t-1}. We model the problem as a stochastic dynamic program ([2]) via the following functional equation,

$$C_t(i, I_{t-1}) = \min_{Q_t \geq 0}\{f_t(i, I_{t-1}, Q_t) + E[C_{t+1}(i_{t+1}, I_{t-1} + Q_t - d_t)|i_{t-1} = i]\}, \quad \text{t=1,\ldots,T-1} \tag{6}$$

where

$$C_T(i, I_{t-1}) = \min_{Q_T \geq 0}\{f_T(i, I_{t-1}, Q_T)|i_{T-1} = i]\} \tag{7}$$

represents the boundary condition.

Example. We illustrate the SDP introduced on the same 4-period example in Section 2. We assume $K = 100$, $h = 1$, $b = 10$, and $c = 1$. We observe that the minimised expected total cost is 262.60 when the opening inventory level is 70.

4 MILP-based models

The (R, S) policy features two control parameters: review periods (R), and order-up-to-levels (S). Under this policy, both R and S are determined at the beginning of the planning horizon; an order is issued to reach the order-up-to-level at the beginning of each review period.

In the literature, [11] built an MILP model upon the piecewise linearisation approach for the first order loss function $L(x, \omega)$ and its complementary function $\hat{L}(x, \omega)$, where ω represents an independent random variable with the probability density function $g_\omega(\cdot)$ and x denotes a scalar variable. Consider a partition of the support Ω of ω into W disjoint compact subregions $\Omega_1, \ldots, \Omega_W$. By fixing a priory the probability mass $p_i = Pr\{\omega \in \Omega_i\}$, the associated conditional expectation E$[\omega|\Omega_i]$ are determined. Based on Jensen's and Edmundson-Madanski inequalities, the first order loss function and its complementary function are approximated with piecewise linear functions $(\sum_{i=1}^W p_i L(x, E[\omega|\Omega_i]), \sum_{i=1}^W p_i \hat{L}(x, E[\omega|\Omega_i]))$. For a special case of standard normally distributed random variables, all p_i and $E[\omega|\Omega_i]$ are precomputed in [13].

We now consider a correlated random variable d_t, for $t = \{1, \ldots, T\}$, we can compute the conditional distribution $f_{d_t|i_{t-1}}(\cdot)$ of $d_t|i_{t-1}$ based on Lemma 1. We apply the piecewise linear approximation proposed in [13] on its conditional distribution. Therefore, $L(x, d_t|i_{t-1})$ and $\hat{L}(x, d_t|i_{t-1})$ are approximated by $\sum_{i=1}^W p_i L\big(x, E\big[\{d_t|i_{t-1}\}|\Omega_i\big]\big)$ and $\sum_{i=1}^W p_i \hat{L}\big(x, E\big[\{d_t|i_{t-1}\}|\Omega_i\big]\big)$, respectively.

Example. We illustrate the MILP model introduced on the same example in Section 3. We observe that the minimum expected total cost is 256.07, when the opening inventory level is 70. Specifically, the reviewing time periods are 1 and 3, and the corresponding order-up-to-levels are 69.18 and 114.34.

5 Conclusion

In this paper we presented a MILP-based model for approximating optimal (R, S) policy parameters with correlated demand. This model is based on a mathematical programming model that can be solved by using-off-the-shelf optimization packages. Our preliminary results show that the optimality gap of our model is tighter, and the computational time of our model is reasonable. This model also can be extended to cover time-series-based demand process, such as AR, MA, ARMA, ARCH.

References

[1] Ronald G Askin. A procedure for production lot sizing with probabilistic dynamic demand. *Aiie Transactions*, 13(2):132–137, 1981.

[2] Richard Bellman. Dynamic programming. *Princeton University Press*, 89:92, 1957.

[3] James H Bookbinder and Jin-Yan Tan. Strategies for the probabilistic lot-sizing problem with service-level constraints. *Management Science*, 34(9):1096–1108, 1988.

[4] Fangruo Chen and Jing-Sheng Song. Optimal policies for multiechelon inventory problems with markov-modulated demand. *Operations Research*, 49(2):226–234, 2001.

[5] Feng Cheng and Suresh Sethi. Optimality of state-dependent (s, S) policies in inventory models with markov-modulated demand and lost sales. 1999.

[6] Awi Federgruen and Paul Zipkin. An efficient algorithm for computing optimal (s, S) policies. *Operations research*, 32(6):1268–1285, 1984.

[7] Stergios Fotopoulos, Min-Chiang Wang, and S Subba Rao. Safety stock determination with correlated demands and arbitrary lead times. *European Journal of Operational Research*, 35(2):172–181, 1988.

[8] Stephen C Graves. A single-item inventory model for a nonstationary demand process. *Manufacturing & Service Operations Management*, 1(1):50–61, 1999.

[9] Gordon Duane Johnson and HE Thompson. Optimality of myopic inventory policies for certain dependent demand processes. *Management Science*, 21(11):1303–1307, 1975.

[10] Grzegorz Reyman. State reduction in a dependent demand inventory model given by a time series. *European journal of operational research*, 41(2):174–180, 1989.

[11] Roberto Rossi, Onur A Kilic, and S Armagan Tarim. Piecewise linear approximations for the static–dynamic uncertainty strategy in stochastic lot-sizing. *Omega*, 50:126–140, 2015.

[12] Roberto Rossi, S Armagan Tarim, Brahim Hnich, and Steven Prestwich. A global chance-constraint for stochastic inventory systems under service level constraints. *Constraints*, 13(4):490–517, 2008.

[13] Roberto Rossi, S Armagan Tarim, Steven Prestwich, and Brahim Hnich. Piecewise linear lower and upper bounds for the standard normal first order loss function. *Applied Mathematics and Computation*, 231:489–502, 2014.

[14] Herbert Scarf. The optimality of (s, S) policies in the dynamic inventory problem. 1959.

[15] Jing-Sheng Song and Paul Zipkin. Inventory control in a fluctuating demand environment. *Operations Research*, 41(2):351–370, 1993.

[16] S Armagan Tarim, Mustafa K Dogru, Ulaş Özen, and Roberto Rossi. An efficient computational method for a stochastic dynamic lot-sizing problem under service-level constraints. *European Journal of Operational Research*, 215(3):563–571, 2011.

[17] S Armagan Tarim and Brian G Kingsman. The stochastic dynamic production/inventory lot-sizing problem with service-level constraints. *International Journal of Production Economics*, 88(1):105–119, 2004.

[18] S Armagan Tarim and Brian G Kingsman. Modelling and computing (R_n, S_n) policies for inventory systems with non-stationary stochastic demand. *European Journal of Operational Research*, 174(1):581–599, 2006.

[19] S Armagan Tarim and Barbara M Smith. Constraint programming for computing non-stationary (R, S) inventory policies. *European Journal of Operational Research*, 189(3):1004–1021, 2008.

[20] Arthur F Veinott Jr. Optimal policy for a multi-product, dynamic, nonstationary inventory problem. *Management Science*, 12(3):206–222, 1965.

Dantzig-Wolfe reformulations for multi-item lot-sizing problems with inventory bounds

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Abstract

We study the *multi-item uncapacitated lot-sizing problem with inventory bounds* (MI-ULS-IB). We present a new MILP formulation for the case of non-speculative (Wagner-Whitin) cost structure using a set of variables to determine the production intervals for each item. Several Dantzig-Wolfe (DW) reformulations of this new formulation are presented and analyzed. These reformulations exploit the structure of the MI-ULS-IB by decomposing it into two subproblems: one relating to the production decisions per item and another that relates to the inventory decisions per period. We propose stabilized column generation algorithms for solving the DW reformulations. Computational experiments are performed to evaluate the proposed formulations and algorithms on a set of benchmark instances involving up to 50 items and 50 periods.

1 Introduction

In various types of production systems and industries it is common to find that inventory levels of products are bounded. These restrictions on the quantities to be stored may be related to physical warehouse space and even to administrative policies, specially for voluminous products, or products requiring special warehouse conditions (i.e., clean rooms, controlled temperatures) [1]. Storage capacity (inventory bounds, IB) considerations are even more relevant for multi-item production structures, where different types of products share storage space. We study the *multi-item uncapacitated lot-sizing problem with inventory bounds* (MI-ULS-IB), a problem of special theoretical and practical interest. General production-distribution planning problems considering IB can be found in [2] and [3]. An industrial application example of a multi-item replenishment-storage planning problem with IB was presented by Akbalik *et al.* [4]. Gutiérrez *et al* [5] presented a variant of the problem with different item weights (or volume), where the bounds are imposed on the total weight of the stock. More formally, the MI-ULS-IB was studied by Akbalik *et al.* [1]. The authors showed that the problem is $\mathcal{NP}$-hard, even for the case of Wagner-Within cost structure. Most recently, Melo and Ribeiro [6] presented a shortest path formulation and a formulation based on the addition of (l,S)-inequalities. The authors also proposed a rounding and relax-and-fix heuristics.

2 Problem statement

The MI-ULS-IB can be described as having m different items to be produced over a finite planning horizon of n periods to satisfy the demand d_t^i for each item $i = 1, \ldots, m$ and each period $t = 1, \ldots, n$ (we assume backlogging is not allowed). Any produced units that are not immediately used to satisfy demand at period t are inventoried in a common storage space. The total amount of inventory in period t is limited by the storage capacity u_t (considering that any item consumes one unit of storage capacity). Producing an item i in any period t incurs a fixed setup cost q_t^i and a variable production cost p_t^i (joint setup costs are not considered). In addition, a holding cost h_t^i is incurred for each unit of item i in stock between period t and $t+1$. We assume no initial and final stocks and nonnegative demands and costs.

To formulate the MI-ULS-IB using the facility location approach for LS problems, let variables w_{lt}^i represent the amount of item i, measured as a fraction of the demand d_t^i, that is produced in period l to satisfy demand of period t, and $y_t^i = 1$ if and only if there is production of item i in period t. Let also I and T be the set of all items and all periods, respectively. The facility location MILP formulation is:

$$\textbf{\textit{FLF}} \quad \text{minimize} \quad \sum_{i=1}^{m}\sum_{t=1}^{n}\left(\sum_{l=t}^{n} c_{tl}^i w_{tl}^i + q_t^i y_t^i\right) \tag{1}$$

$$\text{subject to} \quad \sum_{k=1}^{t} w_{kt}^i = 1 \qquad i \in I,\ t \in T \tag{2}$$

$$w_{kt}^i \leq y_k^i \qquad i \in I,\ k \in T,\ t \in T \tag{3}$$

$$\tag{4}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{t}\sum_{l=t+1}^{n} d_l^i w_{kl}^i \le u_t \qquad 1 \le t \le n-1 \tag{5}$$

$$w_{kt}^i \ge 0 \qquad i \in I,\ k \in T,\ t \in T,\ k \le t \tag{6}$$

$$y_t^i \in \{0,1\} \qquad i \in I,\ t \in T \tag{7}$$

where $c_{tl}^i = d_t^i \left(p_t^i + \sum_{r=t}^{l-1} h_r^i \right)$.

We propose an alternative MILP formulation for the case of non-speculative costs (also known as Wagner-Whitin costs) where producing and storing one unit in a period costs more than producing it later, that is $p_t^i + h_t^i \ge p_{t+1}^i$ for any item i in any period t. This cost structure is very frequent in practical situations and appears in a vast set of lot-sizing literature. This alternative formulation extends the binary variables y_t^i to determine the production time intervals $[k,t]$, $1 \le k \le t \le n$, for each item and are defined as $y_{kt}^i = 1$ is and only if there is production of item i to cover all demand from period k to t. The alternative MILP formulation for the MI-ULS-IB is:

$$\textbf{\textit{CDF}} \quad \text{minimize} \quad \sum_{i=1}^{m}\sum_{k=1}^{n}\sum_{t=k}^{n} q_k^i y_{kt}^i \tag{8}$$

$$\text{subject to} \quad \sum_{k=1}^{t}\sum_{l=t}^{n} y_{kl}^i = 1 \qquad i \in M,\ t \in N \tag{9}$$

$$\sum_{i=1}^{m}\sum_{l=t+1}^{n}\sum_{k=1}^{t} D_{tl}^i y_{kl}^i \le u_t \qquad t \in N \tag{10}$$

$$y_{kt}^i \in \{0,1\} \qquad i \in I,\ k \in N,\ t \in N,\ k \le t \tag{11}$$

where $D_{tl}^i = \sum_{k=t+1}^{l} d_k^i$. For all our preliminary computational experiments we replicate the same instances used by Melo and Ribeiro [6]. They consider neither production costs nor storage costs, which is why the *CDF* only optimizes the setup costs. However, production and storage costs can be easily incorporated.

3 Dantzig-Wolfe reformulations and column generation

Using a reduced version of *CDF*, the MI-ULS-IB can be decomposed into two independent subproblems, one which constitutes an uncapacitated lot-sizing problem (ULS) for $i \in M$, and one which constitutes a special case of a multi-item knapsack problem (CAP) for $t = 1, \ldots, n-1$. ULS makes sure that demand d_t^i is satisfied for every $i \in M$ and $t \in N$, and CAP makes sure that inventory bounds u_t are satisfied for every $t \in N$.

Let U_i denote the subset of configurations for the ULS subproblem for $i \in M$ and C_t denote the subset of feasible configurations for the CAP subproblem for $t = 1, \ldots, n-1$.

The RMP can be stated as follows:

$$\textbf{\textit{RMP}} \quad \text{minimize} \quad \sum_{i \in M} \sum_{c \in U_i} F_i^c L_i^c \tag{12}$$

$$\text{subject to} \quad \sum_{c \in U_i} L_i^c = 1 \qquad i \in M \tag{13}$$

$$\sum_{c \in C_t} X_t^c = 1 \qquad 1 \leq t \leq n-1 \tag{14}$$

$$\sum_{c \in U_i : \sum_{k=1}^{t} \overline{z}_{kl}^i} L_i^c = \sum_{c \in C_t : \overline{y}_{tl}^i = 1} X_t^c \qquad i \in M, \quad 1 \leq t \leq n-1, t+1 \leq l \leq n \tag{15}$$

There are two pricing problems, each associated with one of the subproblems (and each of the two sets of variables in RMP).

References

[1] Akbalik, A., B. Penz, and C. Rapine, Multi-item Uncapacitated Lot Sizing Problem with Inventory Bounds, *Optimization Letters*, 9, 143–154 (2015)

[2] Park, Y.-B., An integrated approach for production and distribution planning in supply chain management, *International Journal of Production Research*, 43, 1205–1224 (2005)

[3] Melo, R., and L.-A. Wolsey, MIP formulations and heuristics for two-level production-transportation problems, *Computers & Operations Research*, 39, 2776–2786 (2012)

[4] Akbalik, A., S. Kebe, B. Penz, and N. Sbihi, Exact methods and a heuristic for the optimization of an integrated replenishment-storage planning problem, *International Transactions in Operational Research*, 15, 195–214 (2008)

[5] Gutierrez, J., M. Colebrook, B. Abdul-Jalbar, and J. Sicilia, Effective replenishment policies for the multi-item dynamic lot-sizing problem with storage capacities, *Computers & Operations Research*, 40, 2844–2851 (2013)

[6] Melo, R.-A., and Celso C. Ribeiro, Formulations and heuristics for the multi-item uncapacitated lot-sizing problem with inventory bounds, *International Journal of Production Research*, 55, 576–592 (2017)

Two-Period Relaxations for Lot-Sizing Problems with Big Bucket Capacities: Consecutive versus Non-Consecutive Two-Period Relaxations

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Abstract

In this study, we investigate two-period relaxations for lot-sizing problems with big bucket capacities. In particular, we study the polyhedral structure of the mixed integer sets related to two-period relaxations where setup times are set to zero. We present two families of strong valid inequalities for two-period relaxations and discuss the separation problems associated with these valid inequalities. Finally, we report preliminary computational results based on generating consecutive two-period relaxations versus generating non-consecutive two-period relaxations for big bucket lot-sizing problems with zero setup times.

1 Introduction

Production planning problems have been interesting for both researchers and practitioners for more than 50 years. The problem aims to determine a plan for how much to produce and stock in each time period during a time interval called planning horizon. It is an important challenge for manufacturing companies because it has a strong impact on their performance in terms of customer service quality and operating costs. In this study, we focus on multi-level, multi-item production planning problems with big bucket capacities, i.e., each resource is shared by multiple items and hence different items can be produced in a specific time period. These real-world problems remain challenging to solve to optimality as well as to obtain strong bounds.

Let NT, NI and NK be the number of periods, items, and machine types, respectively. We assume that each machine type operates only on one level, and each level can employ a number of machine types. The set *endp* indicates all end-items, i.e. items with external demand; the other items are assumed to have only internal demand. Let x^i_t, y^i_t, and s^i_t represent production, setup, and inventory variables for item i in period t, respectively. The setup and inventory cost coefficients are indicated by f^i_t and h^i_t for each period t and item i. The parameter $\delta(i)$ represents the set of immediate successors of item i, and the parameter r^{ij} represents the number of items required of i to produce one unit of item j. The parameter d^i_t denotes the demand for end-product i in period t, and $d^i_{t,t'}$ is the total demand between t and t'. The parameter a^i_k represents the time necessary to produce one unit of i on machine k, and ST^i_k is the setup time for item i on machine k, which has a capacity of C^k_t in period t. Let M^i_t represent the maximum number of item i that can be produced in period t. Following the notation of [2], the multi-level, multi-item production planning problems with big bucket capacities can then be formulated:

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{NT}\sum_{i=1}^{NI} f^i_t y^i_t + \sum_{t=1}^{NT}\sum_{i=1}^{NI} h^i_t s^i_t \\
s.t. \quad & s^i_{t-1} + x^i_t = s^i_t + d^i_t, && t \in [1, NT], i \in endp, && (1) \\
& s^i_{t-1} + x^i_t = s^i_t + \sum_{j \in \delta(i)} r^{ij} x^j_t, && t \in [1, NT], i \in [1, NI] \setminus endp, && (2) \\
& \sum_{i=1}^{NI} (a^i_k x^i_t + ST^i_k y^i_t) \le C^k_t, && t \in [1, NT], k \in [1, NK], && (3) \\
& x^i_t \le M^i_t y^i_t, && t \in [1, NT], i \in [1, NI], && (4) \\
& y \in \{0,1\}^{NT \times NI}, x \ge 0, s \ge 0. && && (5)
\end{aligned}
$$

Here, (1) and (2) are flow conservation constraints for end-items and intermediate items respectively. The constraints (3) are the big bucket capacity constraints, and (4) guarantee that the setup variable is equal to 1 if production occurs. Finally, (5) give the integrality and non-negativity constraints.

We note that uncapacitated relaxation and single-item relaxation have been studied previously by [5]. In addition, [4] introduced and studied the single-period relaxation with preceding inventory, where they also derived cover and reverse cover inequalities for this relaxation. Finally, we also remark the work of [3] on a single-period relaxation as a relevant study.

2 Two-Period Relaxation

Now, we present the feasible region of a two-period, single-machine relaxation of the multi-level, multi-item production planning problems with big bucket capacities, denoted by X^{2PL} (see [1] for details).

$$x^i_{t'} \le \widetilde{M}^i_{t'} y^i_{t'}, \qquad i \in \{1, \dots, NI\}, t' = 1, 2,$$

$$
\begin{aligned}
& x^i_{t'} \leq \widetilde{d}^i_{t'} y^i_{t'} + s^i, && i \in \{1,\dots,NI\}, t' = 1,2, \\
& x^i_1 + x^i_2 \leq \widetilde{d}^i_1 y^i_1 + \widetilde{d}^i_2 y^i_2 + s^i, && i \in \{1,\dots,NI\}, \\
& x^i_1 + x^i_2 \leq \widetilde{d}^i_1 + s^i, && i \in \{1,\dots,NI\}, \\
& \sum_{i=1}^{NI} (a^i x^i_{t'} + ST^i y^i_{t'}) \leq \widetilde{C}_{t'}, && t' = 1,2, \\
& x \geq 0, s \geq 0, y \in \{0,1\}^{2\times NI}.
\end{aligned}
$$

Since we consider a single machine, we dropped the k index from this formulation, however, all parameters are defined in the same lines as before. Observe that for a given time period t, the obvious choice for the "horizon" of this two-period relaxation would be $t+1$, i.e., $t' = 1,2$ relate to the periods of $t, t+\alpha$ with $\alpha \in \{1,\dots,NT-t\}$. The parameters can be associated with the original problem parameters using the relations $\widetilde{M}^i_{t'} = M^i_{t+(t'-1)\alpha}$, $\widetilde{C}_{t'} = C^k_{t+(t'-1)\alpha}$, and $\widetilde{d}^i_{t'} = d^i_{t+(t'-1)\alpha,t+\alpha}$ for all i and $t' = 1,2$.

Next, we remark the following polyhedral result for X^{2PL} (see [1] for details).

Proposition 2.1 *Assume that* $\widetilde{M}^i_t > 0, \forall t \in \{1,\dots,NT\}, \forall i \in \{1,\dots,NI\}$ *and* $ST^i < \widetilde{C}_t, \forall t \in \{1,\dots,NT\}, \forall i \in \{1,\dots,NI\}$*. Then conv(*$X^{2PL}$*) is full-dimensional.*

In this study, we investigate the case of setup times $ST^i = 0, \forall i \in \{1,\dots,NI\}$. Then a promising relaxation of X^{2PL} is established and its polyhedral structures is studied. We derive two classes of valid inequalities, cover and partition inequalities, for this mixed integer set that are valid for X^{2PL} as well. Next, we discuss on the separation problems associated to those valid inequalities. Lastly, we include these cuts in the cutting-planes algorithm to test the effectiveness of these inequalities in closing the integrality gap by generating consecutive two-period relaxation versus generating non-consecutive two-period relaxations. This talk will cover the details of these aspects.

References

[1] M. Doostmohammadi, K. Akartunali, A Polyhedral Study of Two-Period Relaxations for Big-Bucket Lot-Sizing Problems: Zero Setup Case, Technical Report, Dept. of Management Science, University of Strathclyde (2015)

[2] K. Akartunali and A.J. Miller, A computational analysis of lower bounds for big bucket production planning problems, Computational Optimization and Applications, 53, 729-753 (2012)

[3] R. Jans and Z. Degraeve, Improved lower bounds for the capacitated lot sizing problem with setup times, Operations Research Letters, 32, 185-195 (2004)

[4] A.J. Miller, G.L. Nemhauser and M.W.P. Savelsbergh, Solving multi-item capacitated lot-sizing problems with setup times by branch-and-cut, Technical report CORE DP 2000/39, Université Catholique de Louvain, Louvain-la-Neuve (2000)

[5] Y. Pochet, Valid inequalities and separation for capacitated economic lot-sizing, Operations Research Letters, 7, 109-116 (1988)

An Approach for Operational Production and Capacity Planning in Semiconductor Manufacturing

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1 Introduction

Manufacturing planning problems answer very diverse questions on different scales. They range from long-term issues such as the choice of new plants to more operational problems such as the scheduling of operations on machines. Between these two extremes, tactical planning aims at taking mid-term decisions such as determining production plans, i.e. quantities to produce over a time horizon, generally taking into account capacity constraints and customer demands. Capacity planning tries to determine how many tools are required to support a given demand or how much can be produced for a given set of machines while maintaining acceptable performances. Production and capacity planning have been extensively studied for many manufacturing systems for decades [1].

The semiconductor manufacturing industry is known to involve many various, complex and large optimization problems. For an exhaustive review of planning studies in this context, we invite readers to refer to [2]. Regarding production and capacity planning problems, the main sources of difficulty come from the complexity of production flows and the size of problems. Indeed, semiconductor manufacturing plants can be considered as job shop systems including, in particular, re-entrant flows, batching constraints, numerous products and non-identical machines. Furthermore, production factories usually process hundreds of jobs at the same time, all in different stages of their manufacturing routes, each job requiring hundreds of process operations. This complexity forces researchers to develop simplified models, e.g. considering capacity only on bottleneck resources [3], grouping equipment as workcenters [4], or reasoning only on quantities [5]. Our goal is to develop an approach that can handle the complexity of a semiconductor manufacturing facility (fab), and in particular that can consider capacity constraints for all machines and avoid important aggregation choices.

2 Problem Definition

The planning horizon is decomposed into periods, which can be days, weeks or months. A set of jobs currently in the fab (usually called Work In Progress), as well as other jobs to be started are considered. Each job has a customer delivery date also called due date. The objective to optimize can take several forms such as minimization of tardiness, minimization of cycle time or machine use rate maximization. A solution plan has to define, for each operation of each job, the period in which it has to be executed. A number of constraints are considered, the most important ones being capacity constraints. Storage constraints are not considered as they are not critical in semiconductor manufacturing.

Important characteristics of our problem include considering job units (lots of usually 25 wafers), not just volumes, while not trying to determine a detailed schedule. This choice allows a reasonably detailed plan to be determined and larger problems than those encountered in classical operational planning to be considered.

3 Heuristic based planning approach

As previously mentioned, semiconductor manufacturing involves handling problems with hundreds of jobs, each of them requiring hundreds of process operations. Therefore, it is highly unlikely that exact solving methods could be industrially implemented for real life instances. Hence, we developed a planning tool based on the use of a heuristic, which is decomposed into three main modules detailed below.

3.1 Job Projection

This first module requires the input, for every type of product, of the associated theoretical cycle time, which is mainly based on historical data. At the end of this process, the projection module provides theoretical start and end dates for every operation of every job. It consequently generates a forecast of the operations to be processed in each period. Questions such that in which period to assign an operation starting in one period and ending in another have to be answered.

3.2 Workload Balancing

The second module aims at estimating the workload for each machine in each period, i.e. at distributing the product quantities on non-identical machines. To solve this problem, for each period and for each independent group of machines, we solve a linear program whose objective is to minimize the maximum workload of the machines. At the end of this process, we get a unique solution indicating the expected workload for each machine. This workload can be larger than the capacity of the machine, i.e. the machine can be overloaded. Analyzing this information allows potential bottleneck machines to be predicted in the context of infinite capacity planning. To consider the machine capacity constraints, we developed a third module that can be seen as a forward smoothing procedure to move operations from one period to the next.

3.3 Step Shifting

The main input data of this module are the machine workloads. An iterative smoothing procedure is then executed that, from the first period to the last, ensures that the capacity constraints are satisfied. In each period, as long as there are overloaded machines, the procedure selects the most overloaded machine, and among its assigned operations, selects the one associated to the job which is late the least. The selected operation is then postponed (as well as all following operations of the job) to the next period and the corresponding workload is removed from all the machines to which the operation was assigned.

Once the capacity constraints are satisfied for all machines in a period, the projection module is launched again for the next period, taking into account the operations postponed by the "Step Shifting" module. The heuristic ends once all periods have been considered.

The final production plan is the outcome of this module.

4 Performance Evaluation and Perspectives

We also developed a mathematical model to empirically analyze the problem complexity. In order to improve the resolution speed of the model by a standard solver, several techniques were used, such as the aggregation of certain operations or the use of lower bounds established from the study of relaxed problems. However, in spite of these improvements, several hours are still necessary to solve problems with only

10 jobs. Therefore, it is impossible to use this model for problems with hundreds of jobs, let alone thousands.

The heuristic is fully implemented in a factory of STMicroelectronics in France. The decision support tool provides production and capacity plans in one minute for large-sized real problems. A comparative study between the mathematical model and the heuristic for instances of small sizes will be presented in the workshop.

Amongst future studies, extending the heuristic to consider multiple periods in the "Workload Balancing" and "Step Shifting" modules seems promising. Indeed, these modules take greedy decisions in a single period, without analyzing how the workload could be better balanced between periods. For example, considering multiple periods in the "step shifting" module would allow to better choose the lots to move to the next period in the forward smoothing approach already implemented, and to develop a backward smoothing approach.

References

[1] Stadtler, H., Kilger, C., Meyr, H., Supply Chain Management and Advanced Planning, Springer Berlin Heidelberg, Berlin (2015)

[2] Moench, L., Fowler, J., Mason, S., Production Planning and Control for Semiconductor Wafer Fabrication Facilities, Springer (2013)

[3] Horiguchi, K., Raghavan, N., Uzsoy, R., Venkateswaran, S., Finite Capacity Production Planning Algorithms for a Wafer Fabrication Facility. International Journal of Production Research, 39, 825-842 (2000)

[4] Kacar, N., Moench, L., Uzsoy, R., Planning Wafer Starts Using Nonlinear Clearing Func-tions: A Large-Scale Experiment. IEEE Transactions on Semiconductor Manufacturing, 26(4), 602-612 (2012)

[5] Hung, Y., Leachman, R., A Production Planning Methodology for Semiconductor Manufacturing Based on Iterative Simulation and Linear Programming Calculations. IEEE Transactions on Semiconductor Manufacturing, 9, 257-269 (1996)

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Machine Flexibility in Lot Sizing Problems

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Abstract

We analyse the value of process flexibility in the context of a deterministic lot sizing problem with backlogging, where several types of products can be made on several alternative resources, like machines or plants. When multiple parallel resources are present, the standard assumption is that each product can be produced on any of the resources, i.e. we have complete resource flexibility. However, it may be very costly in practice to install resources that have complete flexibility, especially if the products are very different. Therefore, it might be interesting to only implement a limited amount of flexibility where each resource can produce only certain types of items. In order to study the value of such process flexibility, we perform several analyses by proposing some chaining configurations and two mathematical models. The first one determines the best chain configuration and the second one determines the best flexibility configuration for a given budget. Our computational results show that the benefits of a limited amount of flexibility depends on the level of capacity and the homogeneity of the scenarios in terms of cost and demand.

1 Introduction

The literature on flexibility covers a wide spectrum of issues ranging from strategic decisions such as capacity planning [1, 2] to detailed operational issues such as the number of tool changes [3]. The concept of process flexibility in a supply chain defines the type of products that can be manufactured on various alternative resources such as plants or machines.

Jordan and Graves [2] analysed the value of manufacturing process flexibility in a stochastic model with a single period and single stage production environment where multiple products can be produced in different capacitated plants. Each plant can be either dedicated to one specific product or flexible to produce several different products. Demand is random and it is possible that some of the demand will be lost if there is insufficient capacity. More flexibility will allow one to satisfy more of the total demand. The main insight from the paper is that almost all of the benefits of total flexibility can be achieved by implementing only a small amount of flexibility, but in a smart way. To analyse the value of process flexibility, [2] introduced the concept of "chaining". A "chain" is a group of 'products and plants which are all connected, directly or indirectly, by product assignment decisions. Within a chain, a path can be traced from any product or plant to any other product or plant via the product assignment links. The intuition behind this concept is easy to grasp. In the stochastic context presented in [2], the longer the chain of products and plants, the greater the opportunities are for shifting capacity for building products with lower than expected demand to those with higher than expected demand.

This chaining principle has, to the best of our knowledge, not yet been explored in a lot sizing context. Therefore, the main objective of this work is to analyse the trade-off between the benefits of process flexibility and its cost in a lot sizing context. More specifically, we analyse the value of process flexibility in the context of the deterministic lot sizing problem with backlogging which consists basically of determining the size of production lots, i.e. the amounts of each product to be produced in each of the periods in the planning horizon, in a way that minimizes total costs, respects the resource availability and meets the known demand of the products.

2 Analysis of Process Flexibility Configurations

We will analyse the concept of process flexibility in a lot sizing context. The base case for the comparison is the case in which each machine is dedicated to exactly one specific product. In the deterministic lot sizing case, the value of flexibility will be apparent if for this base case (i.e., with only dedicated machines), not all of the demand can be satisfied on time leading to costly backorders. In such a case, adding flexibility (i.e. some machines can produce several types of products instead of just one) can decrease the amount of backlog and hence the total cost. The objective of the experiments is to analyse the effect of long chains, such as proposed in [2], and compare this against several other cases such as the base case with only dedicated machines, the case with random flexibility where there is no specific pattern in the augmented flexibility, and the case of total flexibility where each machine can make every product. We also analyse how different parameters such as the introduction of setup times and the heterogeneity in demand and backlog costs have an impact on the value of flexibility.

For each instance, we tested the effect of using different flexibility configurations on the total cost. Figure 1 shows an example with 6 items, 6 machines and 4 of the 5 flexibility configurations that we analyse. The first case (case (a)) is the dedicated case. In cases (b) and (c), we have added additional links to increase the flexibility. The number of additional links (on top of the base case) is equal to the number of items. However, the flexibility was added in different ways. In case (b) we have 3 clusters of 2 machines, whereas in case (c) we have a long chain. The goal is to show the impact of adding the same number of additional links, when these are being added in different ways. In the final case (d), all the flexibility links are present. In the case of 12 and 24 machines this figure is extended in a straightforward way. For the clustered configuration, we have 6 clusters of 2 machines for the 12 machine case, and 12 clusters of 2 machines for the 24 machine case.

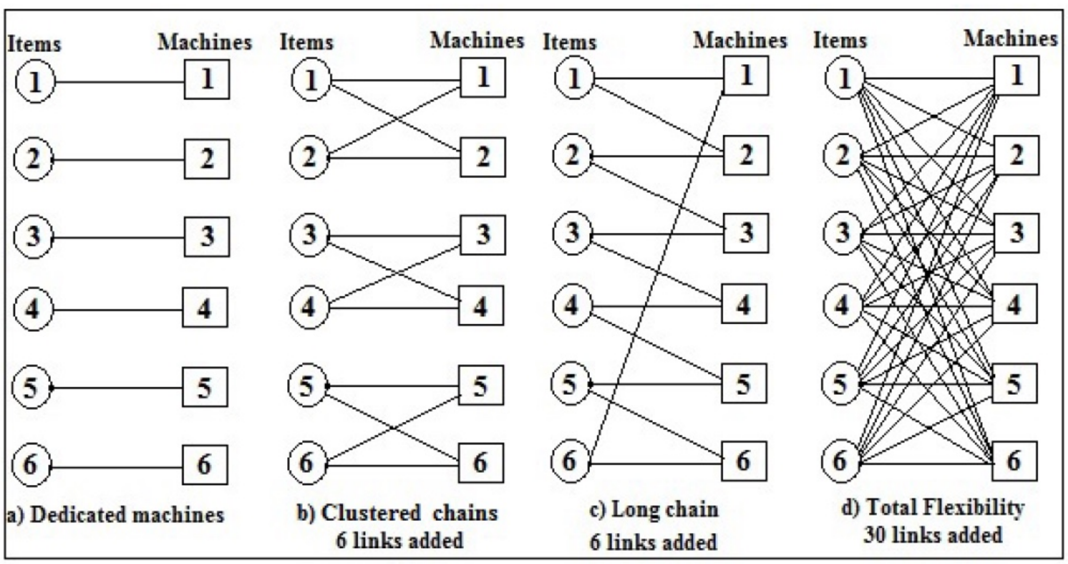

Figure 1: Flexibility configuration for 6 items.

The random flexibility configuration is not presented in Figure 1. In this random flexibility configuration, we add the same amount of links as in the clustered and the long chain, i.e. 6 additional links for the 6 machine case, 12 additional links for the 12 machine case and 24 additional links for the 24 machine case. The additional links are added randomly. Since there are many ways in which links can be added randomly to the dedicated case, we generated 10 different random flexibility configurations. We also note that there are many different long chains and clustered configurations possible, by changing the sequence of the items as explained in Section 3.2. Therefore, we generated 10 different long chains and clustered configurations, by changing the order of the items randomly and keeping the same structure of links between machines and items. Finally, we also propose two mathematical models that allow us to analyse the performance of the best chain and the case where all the product-machine assignments are decision variables.

3 Discussion of the Computational Results

The computational results show that the benefits of flexibility which is measured as the percentage decrease in the objective function value compared to the dedicated case, depends on the characteristics of the instances, but in general are the highest for medium levels of capacity. Furthermore, the benefit of the best chain is very close to the benefit obtained by the total flexibility configuration for high levels of capacity and for homogeneous (cost and demand) scenarios. However, for low capacity levels and for non-homogeneous (cost and demand) scenarios, the best chain has a substantial performance difference compared to the total flexibility case. Moreover, we do see that almost all benefits of process flexibility are found by adding a limited number of links, but not necessarily according to the chain principle and in some cases, this limited number of links is substantially lower than the amount of flexibility needed in a long chain. By comparing the performance of the average long chain with the best chain, we also observe that the exact configuration of the long chain does not matter for the homogeneous case. However, the exact configuration of the chain becomes very important for low capacity levels with backlog heterogeneity and considering setup times, and for medium capacity levels considering demand heterogeneity.

References

[1] Fine C., Freund R., Optimal investment in product-flexible manufacturing capacity. *Management Science*, **36**, 449–466 (1990).

[2] Jordan W. C., Graves S. C., Principles on the benefits of manufacturing process flexibility. *Management Science*, **41(4)**, 577–594 (1995).

[3] Tang C., Denardo E., Models arising from a flexible manufacturing machine, part I: minimization of the number of tool switches. *Operations Research*, **36**, 767–777 (1988).

Capacitated Lot Sizing with a Fixed Product Sequence

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1 Introduction

In some manufacturing systems, the sequence in which products are processed is fixed in order to follow certain production rules, or to minimize setup times and costs such as the ones required when changing colors. However, not all products have to be manufactured in each period. In this context, we consider a special case of the Capacitated Lot Sizing Problem (CLSP) with sequence dependent setups, which is called CLSP with a fixed product sequence. In this case, the number of potential setup sequences is reduced to $O(n2^n)$ compared to $O(n!)$ for the CLSP with sequence dependent setups. To our knowledge, no research has been conducted on this problem. The recent work of [3] considers a fixed sequence in the general job-shop scheduling problem but without sequence dependent setups.

In this presentation, the problem is shown to be NP-hard, and four mixed integer programming formulations are presented. Also, a column generation heuristic is developed. Computational results on benchmark instances are presented to evaluate the proposed formulations and the performance of the column generation approach.

In the CLSP with a fixed product sequence, denoted by CLSP-FS1, the production sequence chosen in each period are restricted to satisfy certain properties, which depend on the fixed product sequence.

Definition 1.1 *Given two sequences $\omega = \langle \omega_1, \omega_2, \ldots, \omega_n \rangle$ and $\alpha = \langle \alpha_1, \alpha_2, \ldots, \alpha_m \rangle$ $(m \leq n+1)$, we say α follows the order of ω, denoted by $\alpha \preceq \omega$, if*

1. *$\alpha_i \in \omega$ for $i \in \{1, 2, \ldots, m\}$.*
2. *$\alpha_i \neq \alpha_j$ for $i \neq j \in \{1, 2, \ldots, m\}$ unless $i = 1$ and $j = m$.*
3. *Let i be the index such that $\omega_i = \alpha_1$ and define sequence $\beta(i) = \langle \omega_i, \omega_{i+1}, \ldots, \omega_n, \omega_1, \omega_2, \ldots, \omega_{i-1}, \omega_i \rangle$. There exists a subset $\Omega' = \{\omega_{i_1}, \omega_{i_2}, \ldots, \omega_{i_{n_m}}\} \subseteq \beta(i)$ such that $\langle \boldsymbol{\alpha_1}, \omega_{i_1}, \omega_{i_2}, \ldots, \omega_{i_{n_1}}, \boldsymbol{\alpha_2}, \omega_{i_{n_1+1}}, \omega_{i_{n_1+2}}, \ldots, \omega_{i_{n_2}}, \boldsymbol{\alpha_3}, \ldots, \boldsymbol{\alpha_m}, \omega_{i_{n_{m-1}+1}}, \ldots, \omega_{i_{n_m}} \rangle = \beta(i)$.*

We consider the CLSP with a single machine whose parameters are:

$\mathcal{N} = \{1, 2, \ldots, N\}$, the set of N products,
$\mathcal{T} = \{1, 2, \ldots, T\}$ the set of T time periods,
cap_t: Machine capacity in period t,
d_{it}: Demand of product i in period t,
pt_{it}: Unitary production time of product i in period t,
hc_{it}: Unitary inventory cost of product i in period t,
b_{it}: Maximum amount of product i that can be produced in period t,
st_{ij}: Setup time from product i to product j,
sc_{ij}: Setup cost from product i to product j,
$\omega = \langle \omega_1, \omega_2, \ldots, \omega_N \rangle$, $\omega_k \in \mathcal{N}$ $\forall k$ and $\omega_i \neq \omega_j$ for $i \neq j$.

In CLSP-FS1, the production sequence and the production quantity of each product in each period must be determined so that all demands are satisfied with a minimum total cost while respecting the machine capacities. Moreover, the chosen period-sequence of each period $s(t)$ has to satisfy that $s(t) \preceq \omega$.

Theorem 1.1 *CLSP-FS1 is NP-hard.*

We prove this theorem by reduction from the knapsack problem.

2 Formulation

We first present an aggregated, sequence-oriented formulation (AG-SO). Let $\mathcal{S} = \{s : s \preceq \omega\}$. Given a sequence $s \in \mathcal{S}$ with length $L(s)$, its associated setup cost $sc(s)$ and setup time $st(s)$ are defined as follows: $sc(s) = \sum_{k=1}^{L(s)-1} sc_{s_k,s_{k+1}}$ and $st(s) = \sum_{k=1}^{L(s)-1} sc_{s_k,s_{k+1}}$. We introduce the following variables for $i \in \mathcal{N}$, $t \in \mathcal{T}$ and $s \in \mathcal{S}$:

$x_{it} \in \mathbb{R}_+$: Quantity of product i produced in period t,
$I_{it} \in \mathbb{R}_+$: Inventory of product i at the end of period t,
$w_{st} \in \{0, 1\}$: Is equal to 1 if sequence s is chosen for period t, and 0 otherwise.

The problem can be formulated as follows:

$$\min \quad \sum_{i\in\mathcal{N},t\in\mathcal{T}} hc_{it} I_{it} + \sum_{s\in\mathcal{S},t\in\mathcal{T}} sc(s) w_{st} \tag{1}$$

$$s.t. \quad I_{i,t-1} + x_{it} = I_{it} + d_{it} \qquad i \in \mathcal{N}, t \in \mathcal{T} \tag{2}$$

$$\sum_{i\in\mathcal{N}} pt_{it} x_{it} + \sum_{s\in\mathcal{S}} st(s) w_{st} \leq cap_t \qquad t \in \mathcal{T} \tag{3}$$

$$x_{it} \leq b_{it} \sum_{s\in\mathcal{S}:i\in s} w_{st} \qquad i \in \mathcal{N}, t \in \mathcal{T} \tag{4}$$

$$\sum_{s\in\mathcal{S}} w_{st} = 1 \qquad t \in \mathcal{T} \tag{5}$$

$$\sum_{s\in\mathcal{S}} s_1 w_{s,t+1} = \sum_{s\in\mathcal{S}} s_{L(s)} w_{st} \qquad t \in \mathcal{T}\backslash\{T\} \tag{6}$$

$$x_{it}, I_{it} \geq 0, \; I_{i,0} = 0 \qquad i \in \mathcal{N}, t \in \mathcal{T} \tag{7}$$

$$w_{st} \in \{0, 1\} \qquad s \in \mathcal{S}, t \in \mathcal{T} \tag{8}$$

The objective function (1) includes the inventory cost and setup costs. The material flow balance constraints are formulated as (2). Constraints (3) ensure that the used capacity does not exceed the available capacity. Constraints (4) express that there can be a production for product i only if there is a setup for i, which implies that a sub-sequence containing i is selected. One and only one sub-sequence can be chosen, which is guaranteed by Constraints (5). At last, Constraints (6) express the consistence of the chosen sequences from one period to the next, which means that the last product of period t should be the same as the first product of period $t + 1$.

We have also developed three other MIP models based on classical formulations for the CLSP ([1]): Aggregated product-oriented formulation (AG-PO), facility location based sequence-oriented formulation (FL-SO) and facility location based product-oriented formulation (FL-PO). The sequence-oriented formulation has exponential size, whereas the size of the product-oriented formulation is polynomial. Due to space limit, they are not presented here.

3 Numerical results

We performed preliminary computational experiments to compare the four formulations on 10 instances from [2] with only the first 10 products and 10 periods. The results obtained with the standard solver IBM ILOG CPLEX 12.6 and a time limit of 10 minutes are summarized in Table 1, which provides average values over all tested instances and for each formulation of: the objective function, the computational time, the relative gap compared to the best known lower bound, the number of columns, the number of binary variables, the number of constraints (rows) and the number of nodes in the search. Moreover, the lower bounds associated to the linear relaxation and its computational time are also given.

Table 1: Computational results

	MIP								LP	
Inst	Obj	Time	Gap (%)	LB	Cols	Binary	Rows	Nodes	Obj	Time
AG-SO	42,921	600	3.8	37,383	102,501	102,300	320	15,448	11,486	6
FL-SO	42,200	601	2.2	40,516	102,851	102,300	770	27,462	39,991	15
AG-PO	42,277	581	2.4	40,612	6,712	6,511	1,312	159,063	9,038	0
FL-PO	42,178	560	2.1	41,188	7,062	6,511	1,762	154,882	37,138	0

* Time in seconds.

Note that formulation FL-PO gives the lowest average gap of (2.1%), whereas formulation AG-SO gives the largest average gap (3.8%). Moreover, formulation FL-PO also has the shortest computational time compared to the other formulations. When considering the linear relaxation, formulation FL-SO gives the best average lower bound while formulation AG-PO gives the worst average lower bound.

References

[1] Pochet, Y., Wolsey, L.A.: Production Planning by Mixed Integer Programming. Springer-Verlag, New York (2006)

[2] Guimãraes, L., Klabjan, D., Almada-Lobo, B., Modeling lotsizing and scheduling problems with sequence dependent setups, European Journal of Operational Research, 239(3), 644–662 (2014)

[3] Wolosewicz, C., Dauzère-Pérès, S., Aggoune, R., A Lagrangian heuristic for an integrated lot-sizing and fixed scheduling problem, European Journal of Operational Research, 244(1), 3–12 (2015)

Lot-sizing models for energy management

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1 Introduction

We address lot-sizing problems in the context of energy management where a single non-reversible energy source is used to fulfill a discrete energy demand over a planning horizon. In addition, a reversible source (such as battery and super-capacitor) can be used to store and/or to supply energy assuming a limited capacity. Therefore, a careful management of the energy storage is required to optimize the total production cost. The non-reversible source is characterized by an efficiency function allowing to get the amount of usable energy for a given cost. The inverse of this function is used in lot-sizing models to get the cost related to the produced amount of energy over the planning horizon. Similarly, efficiency aspects have to be considered for the reversible

source. Losses can also be assumed when carrying energy units from a period to another.
Energy management was already considered within production scheduling problems [1, 2], but very few works address energy management in lot-sizing problems. We quote the work of Masmoudi *et al.* [5] that introduce explicitly the price of electricity as well as availability limitations in the considered lot-sizing models.
The aim of our study is to provide a classification of this new class of lot-sizing problems for energy management mainly based on the structure of the efficiency functions. Our focus is on drawing the boundary between polynomially solvable and NP-complete cases.

2 Problem definition and formulation

We consider that an energy demand d_t in period t has to be satisfied over a discrete planning horizon of T periods. A storage capacity b_t of the reversible source is available in period t. The efficiency function of the non reversible source is denoted by ρ. For the reversible source, f (resp. g) is the efficiency function when the source produces (resp. stores) an amount of energy. The objective function consists in minimizing the total energy cost over the planning horizon to satisfy the demand subject to inventory limitations and efficiency aspects.
The quantity of energy produced by the non reversible source in period t is denoted x_t. The variable s_t is the energy inventory carried from period t to period $t+1$ by the reversible source The amount of energy collected at period t by the reversible source is denoted z_t. The usable energy produced by the reversible source at period t is given by w_t. The mathematical formulation of the problem is provided below.

$$\min \quad \sum_{t=1}^{T} \rho^{-1}(x_t) \tag{1}$$

s.t.

$$x_t - d_t = z_t - w_t \qquad \forall t \in \{1, \cdots, T\} \tag{2}$$

$$s_t = s_{t-1} - f^{-1}(w_t) + g(z_t) \qquad \forall t \in \{1, \cdots, T\} \tag{3}$$

$$s_t \leq b_t \qquad \forall t \in \{1, \cdots, T\} \tag{4}$$

$$x_t, s_t, w_t, z_t \in \mathbb{R}^+ \qquad \forall t \in \{1, \cdots, T\} \tag{5}$$

The objective function (1) minimizes the total energy cost over the planning horizon. Constraints (2) are energy demand constraints where the difference $z_t - w_t$ represents the inventory variation in the reversible source that can be either positive or negative. Constraints (3) are the inventory balance equations and Constraints (4) impose a limitation on the storage capacity of the reversible energy source.

3 Classification and complexity analysis

We introduce a classification based on two fields α/β. They provide the characteristics of the non reversible source efficiency function ($\rho^{-1}(.)$) and the characteristics of the reversible source efficiency functions ($f^{-1}(.)$ and $g(.)$), respectively. The fields α and β can have the following values:

- I: Identity function ($\phi(x) = x$),
- P: Proportional function ($\phi(x) = ax$),
- L: Linear function ($\phi(x) = ax + b$),
- CC: Concave function,
- CV: Convex function,
- PWL: Piecewise linear function.

Table 1 summarizes the complexity results for each configuration of fields α and β when no inventory losses are taken into account. Notations "P", "NP-H" and "?" stand respectively for: Polynomial, NP-Hard and open problem.

		Reversible source efficiency function					
		I	P	L	CC	CV	PWL
Non Reversible source efficiency function	I	P	P	?	?	?	?
	P	P	P	?	?	?	?
	L	P	NP-H	NP-H	NP-H	NP-H	NP-H
	CC	P	NP-H	NP-H	NP-H	NP-H	NP-H
	CV	?	NP-H	NP-H	NP-H	NP-H	NP-H
	PWL	NP-H	NP-H	NP-H	NP-H	NP-H	NP-H

Table 1: Summary of complexity results

A dominance property can be derived regarding the amount of collected and produced energy. We show that there exists an optimal solution that satisfies $w_t z_t = 0$ at each period of the planning horizon. We show that some classes are polynomially solvable such as P/I problem that can be solved in $O(T \log T)$, and L/I and CC/I problems that are solvable in $O(T^2)$. We prove that the PWL/I problem is NP-hard via a reduction from the PARTITION problem and provide a pseudo-polynomial dynamic programming algorithm to solve it. This result generalizes the results of [3]. We also prove that the L/P problem is NP-hard via a reduction from the KNAPSACK problem. We extend some structural properties proposed in [4] to provide a block decomposition of the optimal solutions of the problem.

4 Conclusion and perspectives

In this study, we introduce a new class of lot-sizing problems that deal with energy management. We prove that some classes remain polynomially solvable while several other classes turn to be NP-Hard. We also study some classes when considering inventory losses from one period to another. We show that this problem remains polynomially solvable for the classical CC/I case.
As future research directions, first, it will be interesting to close all open problems in Table 1 and to propose solution methods for each class of problems. A second research direction is to consider inventory losses for all classes addressed in Table 1. A last perspective would be to consider several non reversible sources and/or several reversible sources.

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References

[1] Ngueveu, S. U., Artigues, C., and Lopez, P. (2016). Scheduling under a non-reversible energy source: An application of piecewise linear bounding of non-linear demand/cost functions. *Discrete Applied Mathematics*, 208, 98-113.

[2] Shrouf, F., Ordieres-Mer, J., Garca-Snchez, A., and Ortega-Mier, M. (2014). Optimizing the production scheduling of a single machine to minimize total energy consumption costs. *Journal of Cleaner Production*, 67, 197-207.

[3] Shaw, D. X., and Wagelmans, A. P. (1998). An algorithm for single-item capacitated economic lot sizing with piecewise linear production costs and general holding costs. *Management Science*, 44(6), 831-838.

[4] Atamtürk, A., and Küçükyavuz, S. (2008). An $O(n^2)$ algorithm for lot sizing with inventory bounds and fixed costs. *Operations Research Letters*, 36(3), 297-299.

[5] Masmoudi, O., Yalaoui A., Ouazene Y. and Chehade, H. (2017). Lot-sizing in a multi-stage flow line production system with energy consideration. *International Journal of Production Research*, 55(6), 1640-1663.

Sandwich approximations for lot-sizing problems

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Abstract

We consider single-item lot-sizing problems which are (NP-)hard because of the shape of the objective function, typically not concave. We propose polynomial time approximation algorithms based on a 'sandwich' technique, in which the objective function of the original problem is bounded from below and above by cost objective functions. In fact, finding the tightest sandwich function is an optimization problem on its own, of which the result determines the obtained approximation ratio, typically depending on the problem parameters. We show that this idea can be applied to several lot-sizing problems such as the problem with batch procurement or with a modified all-unit discount cost structure.

1 Introduction

Consider the minimization problem (P)

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & x \in F, \end{array}$$

with $f(x)$ some 'complicated' objective function and F the feasible region. Suppose that we are able to find some 'easy' function $r(x)$ and parameter $\beta > 1$ satisfying

$$r(x) \leq f(x) \leq \beta r(x) \text{ for any } x \in F.$$

Then we say that $f(x)$ is β-sandwiched by $r(x)$. Now consider the relaxed problem

$$\begin{array}{ll} \min & r(x) \\ \text{s.t.} & x \in F, \end{array}$$

and assume it can be solved efficiently with $\hat{x}$ an optimal solution. Then it is not difficult to verify that $\hat{x}$ gives a β-approximation for problem (P). We will apply this principle to several lot-sizing problems.

2 Lot-sizing with batch procurement

Consider some lot-sizing problem with a FTL cost structure with batches of size B. That is, the cost to order a quantity $x \geq 0$ is given by:

$$f(x) = K\mathbb{I}(x > 0) + \lceil x/B \rceil k + px$$

with $K \geq 0$ the setup cost, $k \geq 0$ the fixed cost per batch, $p \geq 0$ the unit ordering cost and $\mathbb{I}(\cdot)$ the indicator function. It is not difficult to verify that for $x \geq 0$ the function $f(x)$ is 2-sandwiched by

$$r(x) = \tfrac{1}{2}(K + k)\mathbb{I}(x > 0) + \tfrac{1}{2}(k + p)x.$$

However, we can find a tighter sandwich function. To this end, consider the parameterized function

$$r_\alpha(x) = (K + \alpha k)\mathbb{I}(x > 0) + ((1 - \alpha)k + p)x$$

with $0 \leq \alpha \leq 1$. If $f(x)$ is β-sandwiched by $r_\alpha(x)$, then one can verify that the conditions $\beta K + \beta\alpha k \geq K + k$ and $\beta(1 - \alpha) \geq 1$ should hold. Therefore, the 'optimal' sandwich function can be found by solving the (non-linear) problem

$$\begin{array}{ll} \min\limits_{\alpha,\beta} & \beta \\ \text{s.t.} & \beta K + \beta\alpha k \geq K + k, \\ & \beta(1 - \beta) \geq 1, \\ & 0 \leq \alpha \leq 1, \end{array}$$

which has an optimal objective value of $\beta^* = \frac{K+2k}{K+k} < 2$ for $K > 0$. For instances such that $K \leq k$, which is a quite realistic assumption, we obtain an a posteriori performance guarantee of at most $3/2$.

We now apply this result to find approximation algorithms for two lot-sizing problems with batch procurement. As the first example, consider the single-item multi-level problem with level-dependent batch sizes B_i, which we can show to be NP-hard.

Now using the above sandwich function for each level i and period t and solving this relaxed problem with the $O(LT^4)$ algorithm of [1], gives a β-approximation with

$$\beta = \max_{i,t} \left\{ \frac{K_t^i + 2k_t^i}{K_t^i + k_t^i} \right\} \leq 2.$$

The second example is a single-level lot-sizing problem where two different batch (or truck) sizes can be used. Each mode i delivers units in batches of size B_i, and incurs a FTL cost. To the best of our knowledge, the complexity of this problem is unknown. Let

$$f_i(x) = K\mathbb{I}(x > 0) + \lceil x/B_i \rceil k_i + p_i x$$

In any period, it is possible to procure units using any combination of the modes. Hence, the optimal cost to procure x units is

$$f(x) = \min_{0 \leq y \leq x} \{f_1(y) + f_2(x - y)\}.$$

Now let $r_i(x)$ be an affine β_i-sandwich function for $f_i(x)$. Then one can show that the function $r(x) = \min_{0 \leq y \leq x}\{r_1(y) + r_2(x - y)\}$ is (i) a β-sandwich function with $\beta = \max\{\beta_1, \beta_2\} \leq 2$ for $f(x)$, and (ii) a concave function. This means that by solving a single-level lot-sizing problem with relaxed cost function $r(x)$ which can be done in $O(T^2)$ time, we obtain a $\max\{\beta_1, \beta_2\}$-approximation algorithm. Similar results hold in case of a fixed number of modes.

3 Modified all-unit discount cost function

Consider a modified all-unit discount cost function with N price rates p_i and discount levels M_i $(i = 1, \ldots, N)$. For suitably chosen M_i' the function is defined as

$$q(x) = \begin{cases} 0 & \text{if } x = 0 \\ C_1 = p_1 M_1 & \text{if } 0 < x \leq M_1 \\ p_1 x & \text{if } M_1 < x \leq M_1' \\ C_2 = p_1 M_1' & \text{if } M_1' < x \leq M_2 \\ p_2 x & \text{if } M_2 < x \leq M_2' \\ \vdots & \\ p_N x & \text{if } x > M_N \end{cases}$$

Note that this is a piecewise linear function with alternating zero and positive slopes. We want to β-sandwich this function $q(x)$ by some affine function $r(x) = a + bx$. In order to optimize the sandwich ratio, it turns out that only the breakpoints are important, and after some further analysis one needs to solve the following non-linear optimization problem:

$$\min_{a,b,\beta} \quad \beta$$

$$\begin{aligned}\text{s.t.}\quad & \beta a \geq C,\\ & \beta(a + bM_i') \geq p_i M_i', \quad \text{for } i = 1, \ldots, N-1,\\ & a + bM_i \leq p_i M_i, \qquad \text{for } i = 1, \ldots, N,\\ & \beta b \geq p_N.\end{aligned}$$

This problem can be solved by realizing that there exists an optimal solution such that (a) $r(x)$ intersects $q(x)$ at least once, (b) $\beta r(x)$ intersects $q(x)$ at least once, and (c) there will be (at least) three intersections in total. This means we can solve the problem by (i) setting three constraints at equality, (ii) solve for a, b, β, and (iii) pick the best among the feasible solutions.

We apply this approach to the single-item problem with modified all-unit discount cost functions, which is NP-hard as shown in [2], who also give a $4/3$-approximation algorithm. The pictures below show our approach applied to one of the examples in this paper. It turns out that two of the four cases lead to a feasible solution with $\beta = 5/3$ the best ratio. Finally, note that our approximation would not become worse (in general better) if we generalize the problem to time-invariant capacities, because the sandwich ratio optimization problem becomes less constrained.

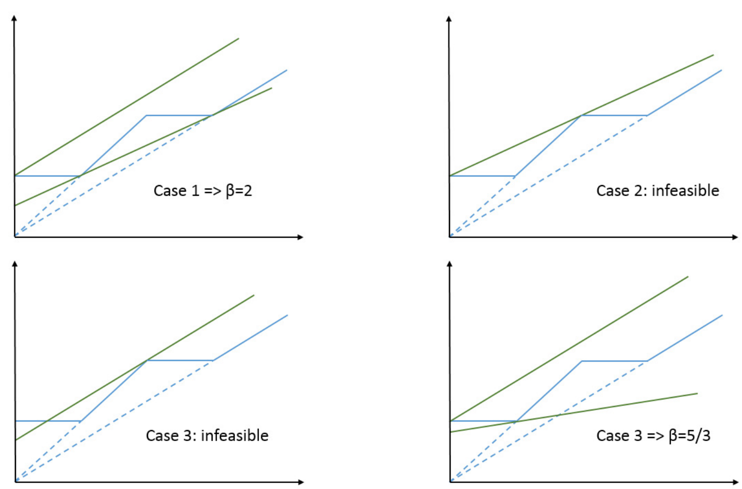

Figure 1: Finding sandwich functions for a modified all-unit discount cost function

References

[1] Zangwill, Backlogging Model and a Multi-Echelon Model of a Dynamic Economic Lot Size Production System–A Network Approach, Management Science, 15, 506-527 (1969)

[2] Chan, Muriel, Shen and Simchi-Levi, On the Effectiveness of Zero-Inventory-Ordering Policies for the Economic Lot-Sizing Model with a Class of Piecewise Linear Cost Structures, Operations Research, 50, 1058-1067 (2002)

Polynomial time algorithms for energy-aware lot sizing problem under stationary energy parameters

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Abstract

We study a single-item lot sizing problem with a limited energy consumption in a shop constituted of M parallel, identical and capacitated machines. Each machine consumes a certain amount of energy when being turned on, when producing units and when being on, whatever it produces or not. In addition to the classical lot sizing costs (setup, unit production and unit inventory costs), we also consider a start-up cost, to pay to turn on a machine. All parameters are time-dependent. The objective is to satisfy the demand over the horizon at minimum cost while respecting the amount of available energy in each period. We show this problem to be NP-hard even under restricted conditions. Then, assuming stationary energy parameters, null setup cost and null running parameters, we propose an exact polynomial time algorithm in $O(M^5T^4)$, with M being the number of machines and T the number of periods. This algorithm is then adapted to solve the general problem in time $O(M^6T^6)$ under stationary energy parameters.

1 Introduction

The optimization of the production planning is no more only cost and benefit oriented, but is also designed to be energy-efficient. Most of the papers published in the domain of energy-aware production planning focus on energy-efficient machine scheduling problems (see Biel and Glock [1]). To the best of our knowledge, there are only a few studies in the literature coupling energy issues with discrete lot sizing problem: Masmoudi *et al.* [3], Giglio *et al.* [2] and Rapine *et al.* [4]. In [3] and [2], the authors consider respectively flow-shop and job-shop systems where they integrate some energy cost or constraints and propose heuristics to solve them. Theoretical results (complexity analysis, exact algorithms, etc.) are thus quasi-inexistant concerning the energy-aware lot sizing problem, apart from Rapine *et al.* [4] detailed below.

The problem we consider here was first introduced by Rapine *et al.* [4]. The authors propose an efficient $O(T \log T)$ time algorithm assuming stationary start-up costs and only one activity (start-up or production) consuming energy. In this study we extend the previous model to a more realistic case. We consider time-dependent cost parameters: Start-up costs, joint production setup costs, and running costs for each machine not turned off. In each period, we have to decide how the available amount of energy is shared among the start-up of the machines, the production of units, and keeping the machines turned on (no matter if producing or not). We show this problem to be NP-hard if some energy parameters are time-dependent, even on a single resource with non-null setup or running costs. We also show that the problem is polynomially solvable if all the energy parameters, that is periodic amount of available energy, start-up and unit consumptions, are stationary. We propose a dynamic programming algorithm in $O(M^6 T^6)$ time for the general case. Assuming stationary energy parameters, null setup cost and null running parameters, the overall complexity is reduced to $O(M^5 T^4)$ time.

2 Problem formulation

A mixed integer linear programming formulation of the problem is given below, with the following decision variables:

x_t :	quantity produced in period t
y_t :	setup variable in period t
s_t :	quantity remainin in stock at the end of t
m_t :	the number of machines running in t
m_t^+ :	the number of machines turned on in t
m_t^- :	the number of machines turned off in t

and the following parameters :

d_t :	demand in period t
U :	capacity of a machine
M :	number of machines in the system
E :	amount of energy available in each period
K_t :	joint set-up cost for producing in period t
c_t :	unit production cost in t
h_t :	unit holding cost from period t to $t+1$
$f_t(k)$:	cost to turn on k machines in t
r_t :	running cost, incurred by each machine running during period t
g_t :	running energy consumption in period t
p_t :	unit energy consumption to produce one unit in t
w_t :	energy consumption to start a machine in t

The MILP formulation of the problem can thus be written as follows :

$$\min \quad \sum_{t=1}^{T}(f_t(m_t^+) + c_t x_t + h_t s_t + r_t m_t + K_t y_t)$$

$$s.t. \quad s_{t-1} + x_t = s_t + d_t \qquad \forall t \in \{1..T\} \quad (1)$$

$$x_t \leq U m_t \qquad \forall t \in \{1..T\} \quad (2)$$

$$p x_t + w m_t^+ + g m_t \leq E \qquad \forall t \in \{1..T\} \quad (3)$$

$$m_t = m_{t-1} + m_t^+ - m_t^- \qquad \forall t \in \{1..T\} \quad (4)$$

$$m_t \leq M \qquad \forall t \in \{1..T\} \quad (5)$$

$$x_t \leq U M y_t \qquad \forall t \in \{1..T\} \quad (6)$$

$$s_t \geq 0, x_t \geq 0, y_t \in \{0,1\}, m_t \in \mathcal{Z}^+, m_t^+ \in \mathcal{Z}^+, m_t^- \in \mathcal{Z}^+ \qquad \forall t \in \{1..T\} \quad (7)$$

Constraint (1) is the material balance between the production, storage and demand in each period. In a period t the production is limited by two constraints: production capacity due to the machines on (2) and energy restriction constraint (3). Constraint (4) represents the total number of machines running in each period. Constraint (5) indicates that the number of machines running can not exceed the total number of machines in the system. Constraint (6) forces variable y_t to be equal to 1 if the quantity x_t is positive. The feasibility domains are given by constraint (7).

We assume that production costs follow non-speculative motives, also called Wagner-Whitin (WW) cost structure.

Theorem 1 *If the number M of machines is part of the instance, problem energy-LSP is NP-hard even with null production cost ($c = 0$) and null holding cost ($h = 0$), and with stationary energy parameters E and w.*

3 Polynomial time algorithms

We propose dynamic programming algorithms to solve *energy*-LSP under stationary energy parameters. We show that, in a dominant solution, each production period with a non null entering stock uses either entirely the available capacity (all the machines that are not turned off produce at full capacity), or entirely the available amount of energy (for production and/or for starting additional machines). In a classical approach, we decompose the problem into subplans, and evaluate the optimal cost of each subplan, for a given number of machines turned on at the beginning and at the end of the subplan. Finally, we use a shortest path algorithm to obtain the overall optimal cost. We obtain the following result:

Theorem 2 *Problem energy-LSP can be solved in polynomial time in $O(M^5T^4)$ if energy parameters p, w and E are stationary.*

Theorem 3 *Problem energy-LSP with running costs, joint setup costs and running energy consumption can be solved in polynomial time in $O(M^6T^6)$ if energy parameters p, w, g and E are stationary.*

References

[1] Biel, K. and Glock, C.H. (2016). Systematic literature review of decision support models for energy-efficient production planning. *Computers and Industrial Engineering*, 101, 243-259.

[2] Giglio, D., Paolucci, M. and Roshani, A. (2017). Integrated lot sizing and energy-efficient job shop scheduling problem in manufacturing/remanufacturing systems *Journal of Cleaner Production*, 148, 624-641.

[3] Masmoudi, O., Yalaoui, A., Ouazene, Y. and Chehade, H. (2017). Lot-sizing in a multi-stage flow line production system with energy consideration. *International Journal of Production Research*, 55 (6), 1640-1663.

[4] Rapine, C., Penz, B., Gicquel, C. and Akbalik, A. (2016). Lot sizing problem with energy constraints. *International Workshop on Lot Sizing (IWLS)*, Hannover, Germany, 2016.

Robust inventory problem revisited

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Abstract

We consider the lot-sizing problem where demands are uncertain. The demand can be satisfied by production, by inventory held in stock or backlogged. A recourse model is considered where the production decisions are first stage decisions and the stock and backlog variables are adjustable to the demands. For the uncertainty set, we consider the classical budget polytope introduced by Bertimas and Sim (2003, 2004).

We revisite the two classical robust approaches for this problem: the true min-max approach introduced by Bienstock and Ozbay (2008) and the dualization approach from Bertsimas and Thiele (2006). Our main contribution is to derive the dualization approach from a lagrangean relaxation of the maximization sub-problem occurring in the min-max approach, and, as a consequence, to provide a better understanding of the relation between the two approaches. Moreover, such relaxation can be regarded as a less conservative and tractable robust approach.

1 Introduction

We consider a simple lot-sizing problem defined for a finite time horizon $T = \{1, \ldots, n\}$. For each time period, $t \in T$, the unit holding cost h_t, the unit backlogging cost b_t and the unit production cost c_t are considered. The demand in time period $t \in T$ is given by d_t. For $t \in \{1, \ldots, n+1\}$, we define x_t as the inventory at the start of period t (x_1 is the initial inventory level). If x_t is negative it indicates a shortage. Variables $u_t \geq 0$ indicate the quantity to produce in time period $t \in T$.

When d_t is known and fixed we obtain a basic deterministic lot-sizing problem that can be modelled as follows:

$$\begin{aligned} \min_{u,x} \quad & \sum_{t=1}^{T} (c_t u_t + \max\{h_t x_{t+1}, -b_t x_{t+1}\}) \\ s.t. \quad & x_{t+1} = x_t + u_t - d_t, && 1 \leq t \leq T, \\ & u_t \geq 0, && 1 \leq t \leq T. \end{aligned}$$

where $\max\{h_t x_{t+1}, -b_t x_{t+1}\}$ gives the holding cost $h_t x_{t+1}$ if $x_{t+1} > 0$ and the backlogging cost $-b_t x_{t+1}$ if $x_{t+1} < 0$ at the end of time period $t+1$.

Here we consider the case where the demands d_t are uncertain and belong to the well known budget polytope introduced by Bertsimas and Sim [3]:

$$\Delta = \{d | d_t = \mu_t + \delta_t z_t, \sum_{j=1}^{t} |z_j| \leq \Gamma_t, \, z_t \in [-1, 1], t \in T\}.$$

Two approaches to handle with demand uncertainty have been proposed. Bienstock and Ozbay (2008) propose a min-max approach where, for a given production vector, the demand d_t is being picked by an adversary. The min-max formulation is the following:

$$R^* = \min_{u \geq 0} R(u)$$

where

$$\begin{aligned} R(u) = \max_{z,x} \quad & \sum_{t=1}^{T} (c_t u_t + \max\{h_t x_{t+1}, -b_t x_{t+1}\}) && && (1) \\ s.t. \quad & x_{t+1} = x_t + u_t - d_t, && t \in T, && (2) \\ & d_t = \mu_t + \delta_t z_t, && t \in T, && (3) \\ & \sum_{j=1}^{t} |z_j| \leq \Gamma_t, && t \in T, && (4) \\ & z_t \in [-1, 1], && t \in T. && (5) \end{aligned}$$

$R(u)$ is known as the adversarial problem. $R(u)$ can be modelled as mixed integer problem as follows.

First, using equations (2) and (3) it is possible to write

$$x_{t+1} = x_1 + \sum_{j=1}^{t} (u_j - \mu_j - \delta_j z_j), \quad t \in T. \qquad (6)$$

Using (6) and setting $y_t = \max\{h_t x_{t+1}, -b_t x_{t+1}\}$, R(u) can be modelled as follows (see [7]):

$$
\begin{aligned}
R(u) = \ & \max_z \sum_{t=1}^{T} (c_t u_t + y_t) && && (7)\\
s.t. \ & y_t \le h_t x_{t+1} + M w_t, && t \in T, && (8)\\
& y_t \le -b_t x_{t+1} + M(1 - w_t), && t \in T, && (9)\\
& w_t \in \{0, 1\}, && t \in T, && (10)\\
& \sum_{j=1}^{t} |z_j| \le \Gamma_t, && t \in T, && (11)\\
& z_t \in [-1, 1], && t \in T. && (12)
\end{aligned}
$$

Bienstock and Ozbay [5] solve the min-max problem using a decomposition approach. A similar approach is used in [1] where the adversarial problem is solved by dynamic program.

Another approach was proposed earlier by Bertsimas and Thiele [5] following the general robust optimization approach proposed in [4]. The idea is to consider for each individual constraint (6) the realization of the uncertainty parameters (demands) leading to the worst case value (higher cost). This leads to the following model (see [5]):

$$
\begin{aligned}
C^* = \min_u \ & \sum_{t=1}^{T} (c_t u_t + y_t) && && (13)\\
s.t. \ & y_t \ge h_t \left(x_1 + \sum_{j=1}^{t} (u_j - \mu_j) + A_j \right), && t \in T && (14)\\
& y_t \ge -b_t \left(x_1 + \sum_{j=1}^{t} (u_j - \mu_j) + A_j \right), && t \in T && (15)\\
& u_t \ge 0, && t \in T. && (16)
\end{aligned}
$$

where

$$
\begin{aligned}
A_t = \max_v \ & \sum_{j=1}^{t} \delta_j z_j \\
& \sum_{j=1}^{t} |z_j| \le \Gamma_t, \quad t \in T \\
& z_t \in [-1, 1], \quad t \in T
\end{aligned}
$$

As the worst case for each time period can occurs for different demand values one can easily see that the Bertsimas and Thiele approach tends to be more conservative than the min-max approach, where the solution for the adversarial problem is a single demand vector. Consequently, $R^* \leq C^*$. On the other hand model (13)-(16) is easier to solve than the min-max problem. In order to avoid both the conservativeness of (13)-(16) and the hardness of the min-max approach, in [2] and [7] are discussed other conservative approximations for the min-max problem obtained through relaxations of the uncertainty set, namely using affine approximations.

Our contribution is to derive a lagrangean relaxation from a reformulation of the adversarial problem (7)-(12). From that relaxation we obtain new conservative approximations for the min-max problem, and in particular, we show the intuitive result that when the multipliers are null model (13)-(16) is obtained. Computational tests are reported to compare the true min-max values with the other conservative approximations.

References

[1] Agra A, Santos M C, Poss M, Nace D, A dynamic programming approach for a class of robust optimization problems, SIAM Journal of Optimization, 26(3), 1799 - 1823, 2016.

[2] Ardestani-Jaafari A, Delage E, Robust optimization of sums of piecewise linear functions with application to inventory problems, Operations Research, 64(2), 474 - 494, 2016.

[3] Bertsimas D, Sim M, Robust discrete optimization and network ows, Mathematical Programming, 98, 49 - 71, 2003.

[4] Bertsimas D, Sim M, The price of robustness, Opererations Research, 52, 35 - 53, 2004.

[5] Bertsimas D, Thiele A, A robust optimization approach to inventory theory, Operations Research, 54(1), 150 - 168, 2006.

[6] Bienstock D, Özbay N, Computing robust basestock levels, Discrete Optimization, 5 , 389 - 414, 2007.

[7] Gorissen B, Hertog D, Robust counterparts of inequalities containing sums of maxima of linear functions, European Journal of Operational Research, 227(1), 30 - 43, 2013.

A decomposition algorithm for the robust lot sizing problem with remanufacturing option

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Abstract

This study presents a decomposition procedure for solving the economic lot sizing problem with remanufacturing under non-probabilistic parameter uncertainties. Our method is motivated by reducing the computational effort required for solving such complex problems, while demand and return parameters are represented as parts of uncertainty sets, using the approach presented in [2]. Our decomposition procedure is a variation of the work of [1], where optimal base stock levels are computed for the traditional lot sizing problem with linear production costs.

1 Introduction

The method presented in this paper is based on production systems with item recoveries, where total production costs and waste are reduced through restoring deformed products to their usable state. More specifically, we are interested in the lot sizing problem with remanufacturing (LSR), where our main aim is to construct an optimal production plan that minimizes the total operational cost under inventory balance and demand satisfaction constraints. Despite the variety of research on traditional lot sizing problems (LS), the remanufacturing option has not been addressed as widely.

Initial studies on LSR problems include the work of [4] and [5], where the well-known Wagner-Whithin algorithm is implemented. The complexity of the LSR problem was shown to be NP-hard by the recent work of [3], motivating further research to investigate various methods to potentially reduce the computational time requirements. A dynamic programming algorithm presented by [6] has shown that the

LSR problem can be solved in $O(T^4)$ time under the absence of variable production costs. [7] have shown the tractability of a polynomial time special case and have introduced two classes of valid inequalities for the capacitated version of the problem. However, studies concerning the influence of uncertainty on these formulations is very scarce, with the exception of the very recent work of [8], looking into some classes of robust problems including lot sizing.

Our approach is mainly motivated by the study of [1], where a decomposition procedure for solving robust inventory problems under demand uncertainty is introduced. The proposed method in this study aims to extend the aforementioned framework to contribute to the growing research on LSR, where demand and return uncertainty are present.

2 Problem Formulation

2.1 Problem Structure

The objective function of our problem minimizes the total costs associated with setup, manufacturing, remanufacturing, inventory, backlogging and disposal of returned items. We assume two different levels of inventory with specific costs: serviceables and returns. The serviceables inventory level applies for all items that are considered as-good-as-new, including the returned items that have been remanufactured, while returns that have not yet been remanufactured are kept as a different inventory level called the returns inventory. In order to produce items, a joint setup cost has to be incurred, meaning that both manufacturing and remanufacturing would be allowed for that particular time period.

2.2 Parameter Uncertainties

We consider parameter realizations to be parts of independent uncertainty sets. These uncertainty sets can be shown as U_t^D for demands and U_t^R for returns on a given time period $t = 1..T$.

All uncertainty sets are formed as budgeted polytopes (see [2]). Accordingly, we represent a given demand (return) realization as $D_t = \overline{D}_t + \hat{D}z_t^D$ ($R_t = \overline{R}_t + \hat{R}z_t^R$), where their exact values are determined by the scaled deviation variables, z_t^D (z_t^R). The values of scaled deviation variables are constrained as $0 \leq z_t^D \leq 1$ and $0 \leq z_t^R \leq 1$ as they represent the relative amount of deviation from maximum allowed deviations $\hat{D}$ and $\hat{R}$. This results in the demands and returns to take on values within the intervals $[\overline{D_t}, \overline{D_t} + \hat{D}_t]$ and $[\overline{R_t}, \overline{R_t} + \hat{R}_t]$, respectively.

3 Decomposition Algorithm

Our decomposition algorithm solves a restricted version of the robust LSR problem iteratively. The restricted problem, which is referred to as the Decision Maker's Problem (DMP), is solved to optimality for a subset of demand $\tilde{U}^D \subseteq U^D$ and return $\tilde{U}^R \subseteq U^R$ points. Once solved, a new demand and return vector is returned by another problem called the Adversarial Problem (AP). The objective of the AP is to seek for new demand and return points that maximize the total inventory and backlogging costs for the production plan that has been generated by the DMP, through implementing different demand and return values from the corresponding non-restricted uncertainty sets (U^D and U^R). Each time a new demand and return vector is returned by the AP, the restricted uncertainty sets are updated and the DMP is re-solved, until there are no remaining parameter realizations that result in the generation of a production plan with a greater total cost. Due to the number of extreme points in the initial uncertainty sets (U_t^D and U_t^R) being finite, the DMP and AP are guaranteed to converge.

References

[1] Daniel Bienstock and Nuri Özbay, Computing robust basestock levels, Discrete Optimization, 5, 389–414 (2008)

[2] Dimitris Bertsimas and Melvyn Sim , The price of robustness, Operations research, 52, 35–53 (2004)

[3] Mathijn J Retel Helmrich and Raf Jans and Wilco van den Heuvel and Albert PM Wagelmans, Economic lot-sizing with remanufacturing: complexity and efficient formulations, IIE Transactions, 46, 67–86 (2014)

[4] Knut Richter and Mirko Sombrutzki, Remanufacturing planning for the reverse Wagner/Whitin models, European Journal of Operational Research, 121, 304–315 (2000)

[5] Knut Richter and Jens Weber, The reverse Wagner/Whitin model with variable manufacturing and remanufacturing cost, International Journal of Production Economics, 71, 447–456 (2001)

[6] Ruud H Teunter and Z Pelin Bayindir and Wilco Van Den Heuvel, Dynamic lot sizing with product returns and remanufacturing, International Journal of Production Research, 44, 4377–4400 (2006)

[7] Kerem Akartunalı and Ashwin Arulselvan, Economic Lot-Sizing Problem with Remanufacturing Option: Complexity and Algorithms, Machine Learning, Optimization, and Big Data, MOD, 132–143 (2016)

[8] A. Agra and M. C. Santos and D. Nace and M. Poss, A dynamic programming approach for a class of robust optimization problems, SIAM Journal on Optimization, 26, 1799–1823 (2016)

[9] K. Akartunalı and I. Fragkos and A.J. Miller and T. Wu, Local Cuts and Two-Period Convex Hull Closures for Big-Bucket Lot-Sizing Problems, INFORMS Journal on Computing, 28, 766–780 (2016)

Optimal policies for production-clearing systems with stochastic demands

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Abstract

We consider a production-clearing system where a product is produced on a single production resource at a constant rate without stopping, and then buffered to meet stochastic demands. The system is unconventional as the production rate is typically higher than the demand rate. We show that an (m, q)-policy, i.e., a policy that clears the buffer to level m as soon as the inventory hits a level q, minimizes the long-term average cost. We also develop a very efficient computational scheme to find the parameters of the optimal policy.

1 Background

The models of production-clearing inventory models are useful to analyze inventory systems where the supply of materials typically exceeds the demand or when it is too expensive or difficult to switch off or reduce the production rate.

There are many industrial examples of such systems. For instance, dairy cooperatives are legally binded to acquire the milk produced in member dairy farms. It is, therefore, not possible to reduce the milk supply when inventory levels are high and/or demand rate is low. Also, in industries such as steel-casting or glass-production the setup costs and/or setup times are so large that switching off continuous production is economically undesirable, despite being possible in principle. Barron (2015) discusses many other interesting examples where production-clearing models are relevant.

It is important to note that the inventory control problem faced in traditional inventory systems is essentially different than the one in production-clearing systems.

The problem in inventory systems is to control the input (in the form of replenishment orders) such that customer demand is met. The problem in production-clearing systems, on the other hand, is to control the output (in the form of clearing actions) as production cannot be controlled.

2 Related Literature

The stochastic production-clearing systems were first in the earlier works of Stidham (1974, 1977, 1986). The contemporary literature on the subject considers generalizations of the models introduced in these classical studies.

We can mention two main production-clearing problems in this line of research which differ with respect to accessibility of the clearing option is over time. First, there are "continuous-review" problems where it is always possible to clear inventories. Second, there are "sporadic-review" problems where clearing is only possible at random moments in time. Because the current study considers a continuous-review problem, in the following we concentrate on the literature in this variant of the problem.

The literature on continuous-review stochastic production-clearing systems predominantly concentrates on computing the performance of a given clearing policy. In this context, Kim and Seila (1993), Perry et al. (2005) and Barron (2015) consider different cost structures and demand processes.

There are only a few studies which aim at finding an optimal policy that minimize the long-term average cost.Berman et al. (2005) consider a production-clearing system with exponentially distributed demand. The system is controlled by an "all-or-nothing" policy that clears all inventory when the inventory level hits or exceeds a critical level q. For this system, they minimize the long-run average cost as a function of the clearing level q. The average cost function is stated in terms of the steady-state distribution of the inventory level, which is obtained by using level crossing arguments. Barron (2015) discusses a production-clearing system where the production switches between a number of rates. However, it is assumed the clearing policy follows an all-or-nothing rule.

3 Overview and Results

We consider a system where a single machine produces at a fixed rate and without stopping a certain product into a buffer. The production rate is constant. The demand is stochastic and follows a compound Poisson process. The production rate is larger than the demand rate. Because production never stops, it is necessary to prevent inventory levels to grow without bound. This is done by means of a clearing policy that occasionally prescribes to clear a part or the complete inventory, for instance by selling (salvaging) the surplus in a secondary market with ample demand. Thus, the control problem involves two types of decisions: when to clear and to what level to clear the inventory. We assume general piecewise continuous inventory costs. The clearing cost has fixed and variable components (c.f., Kim and Seila, 1993). We model demands as a compound Poisson process with a general demand size distribution. Our analysis applies to both backordering and lost sales. We aim at identifying a stationary clearing policy which minimizes the long-run average cost.

The main contributions of this study are as follows. First, we show that an (m, q)-policy, i.e., a policy that clears the buffer to level m as soon as the inventory hits a level q, minimizes the long-run average cost. This policy is more general than the all-or-nothing policies considered in the literature. Second, we derive a numerically efficient approach for finding the average-cost optimal policy. This approach uses the concept of g-revised cost (see e.g. Wijngaard and Stidham, 1986) which is based on the idea of reformulating a stochastic optimization problem into an equivalent optimal stopping problem which is much easier to analyze. Because we consider the stochastic production-clearing systems under very general assumptions, our results and methods subsume most of the earlier findings in the literature. Third, we numerically investigate the sensitivity of the optimal policy. The results of our numerical study reveals that even relatively small deviations from the optimal parameters can lead to a substantial increase in the long-run average costs. This is especially critical when the utilization is high.

References

Barron, Y. (2015). Clearing control policies for MAP inventory process with lost sales. *European Journal of Operational Research 251*, 495–508.

Berman, O., M. Parlar, D. Perry, and M. J. M. Posner (2005). Production/clearing models under continuous and sporadic reviews. *Methodol. Comput. Appl. Probab. 7*, 203–224.

Kim, K. and A. F. Seila (1993). A generalized cost model for stochastic clearing systems. *Comput. Oper. Res. 20*(1), 67–82.

Perry, D., W. Stadje, and S. Zacks (2005). Sporadic and continuous clearing policies for a production/inventory system under an M/G demand process. *Math. Oper. Res. 30*(2), 354–368.

Stidham, S. (1974). Stochastic clearing systems. *Stoch. Process. Appl. 2*(1), 85–113.

Stidham, S. (1977). Cost models for stochastic clearing systems. *Oper. Res. 25*(1), 100–127.

Stidham, S. (1986). Clearing systems and (s, S) inventory systems with nonlinear costs and positive leadtimes. *Oper. Res. 34*(2), 276–280.

Wijngaard, J. and S. Stidham (1986). Forward recursion for Markov decision processes with skip-free-to-the-right transitions, part I: Theory and algorithm. *Math. Oper. Res. 11*(2), 295–308.

Single item stochastic lot sizing problem considering capital flow and business overdraft

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Abstract

This paper introduces capital flow to the single item stochastic lot sizing problem. A retailer can leverage business overdraft to deal with unexpected capital shortage, but needs to pay interest if its available balance goes below zero. A stochastic dynamic programming model maximizing expected final capital increment is formulated to solve the problem to optimality. We then investigate the performance of four controlling policies: (R, Q), (R, S), (s, S) and $(s,\ \overline{Q},\ S)$; for these policies, we adopt simulation-genetic algorithm to obtain approximate values of the controlling parameters. Finally, a simulation-optimization heuristic is also employed to solve this problem. Computational comparisons among these approaches show that policy (s, S) and policy $(s, \overline{Q}, S)$ provide performance close to that of optimal solutions obtained by stochastic dynamic programming, while simulation-optimization heuristic offers advantages in terms of computational efficiency. Our numerical tests also show that capital availability as well as business overdraft interest rate can substantially affect the retailer's optimal lot sizing decisions.

1 Introduction

Overdraft is widely used by many companies to prevent capital shortage. It is necessary and important for a manager to take capital flow and external financing into account when making operational decisions. Our contributions to the lot sizing problem are the following:

- We incorporate capital flow and one kind of external financing, i.e. overdraft, in the stochastic lot sizing problem and formulate a stochastic dynamic programming model to obtain optimal solutions.
- We discuss four inventory controlling policies for this problem and use simulation-genetic algorithm to obtain approximate values of the controlling parameters.
- We introduce a simulation-optimization heuristic inspired by the approach.
- We conduct a comprehensive numerical study to compare stochastic dynamic programming, simulation-genetic algorithm and simulation-optimization heuristic.

2 Problem description

All the notations adopted in this paper is listed in Table 1. In our problem, demand is stochastic and non-stationary. For each period t, its demand is represented by D_t, which is a non-negative random variable with probability density function f_t, cumulative distribution function F_t, mean μ_t, variance σ^2. Random demand is assumed to be independent over the periods. Unmet demand in any given period is back ordered and satisfied as soon as the replenishment arrives. Excess stock is transferred to next period as inventory and the sell back of excess stock is not allowed.

We assume the initial capital quantity of the retailer is B_0; order delivery lead time is zero; selling price of the product is p and the retailer receives payments only when the products are delivered to customers; a fixed cost a is charged when placing orders, regardless of the ordering amount, and R_t is a 0-1 variable to determine whether the retailer makes order at period t; a variable cost v is charged on every ordering unit; end-of-period inventory level for period t is I_t, and we set I_t^+ to represent $\max\{I_t, 0\}$ and I_t^- to represent $\max\{-I_t, 0\}$; a variable inventory holding cost h is charged on every product unit carried from one period to the next; per unit stock-out penalty cost is π; at the beginning of each period t, its present capital is B_{t-1}, if its initial capital is below zero, the retailer has to pay interests with a rate of b.

End-of-period capital B_t for period t is defined as its initial capital B_{t-1}, plus payments by customers for satisfied demand of this period, minus the payments to suppliers for orders made in this period and this period's fixed ordering cost, holding and backorder costs, and minus the interest paid if its initial capital is negative. It can be represented by the following equation.

$$B_t = B_{t-1} + p\min\left\{D_t + I_{t-1}^-, Q_t + I_{t-1}^+\right\} - \left(vQ_t + aR_t + hI_t^+ + \pi I_t^-\right) - b\max\{-B_{t-1}, 0\} \tag{1}$$

The actual sales amount in period t is $\min\left\{D_t + I_{t-1}^-, Q_t + I_{t-1}^+\right\}$, where $D_t + I_{t-1}^-$ is demand plus backorder in period t and $Q_t + I_{t-1}^+$ is the total available stock in period t.

For the final capital of the retailer in the whole planning horizon, we defined it as the end-of period capital B_T, minus the interest paid if B_T is negative, which is:

$$B_{T+1} = B_T - b\max\{-B_T, 0\} \tag{2}$$

We use a tilde above the parameter to represent its expected value. Our aim is to find a replenishment plan that maximizes the expected final capital increment, i.e. $\tilde{B}_{T+1} - B_0$.

3 Results and discussion

6 periods with different demand patterns are adopted for experiments and there are 640 numerical cases in total, our computation results show that policies (s, S) and $(s, \overline{Q}, S)$ solved by genetic algorithm, in general perform better than other approaches (RMSE: 3.17 and 3.25, respectively; MAPE: 5.68% and 5.59%, respectively), followed by policy (R, S) (RMSE: 6.28, MAPE: 26.72%), simulation-optimization heuristic (RMSE: 13.60, MAPE: 53.67%) and policy (R, Q) (RMSE: 15.90, MAPE: 66.63%). Considering the confidence levels, performance of policy (s, S) and policy $(s, \overline{Q}, S)$ are essentially identical. For the four controlling policies, it can be concluded that their performance is related with their flexibility. Since policy (s, S) and policy $(s, \overline{Q}, S)$ are based on "dynamic uncertainty" strategy, which is most flexible, they perform best for the problem, while the least flexible policy (R, Q) has worst performance. It is however surprising that enforcing a maximum order quantity $\overline{Q}$ does not seem to be beneficial, and that an (s, S) policy with parameters carefully selected seems to provide competitive performances.

The performance of different approaches does not seem to be affected by different parameter levels under the criterion RMSE; however, it is affected by the margin of product — selling price and unit variable ordering cost under the MAPE criterion. Finally, the performance of the simulation-optimization heuristic varies substantially across different demand patterns.

In terms of computation times, the simulation-optimization heuristic runs faster than genetic algorithm, with average computation time less than one second (0.04s). Among the policies solved via genetic algorithm, policy (R, S) runs fastest (42.31s), followed by policy (R, Q) (46.91s), policy (s, S) (184.88s), policy $(s, \overline{Q}, S)$ (194.39s).

Notations	Description
Indices	
t	Period index, $t = 1, 2, \ldots, N$
Problem parameters	
B_0	Initial capital
I_0	Initial inventory level
I_t^+	$\max\{I_t, 0\}$
I_t^-	$\max\{-I_t, 0\}$
p	Product selling price
a	Fixed ordering cost
v	Unit variable ordering cost
h	Unit inventory cost
π	Unit penalty cost for back orders
b	Interest rate for minus capital
M	A big number
Random variables	
D_t	Random demand at period t with probability density function $f_t(D_t)$, cumulative distribution function $F_t(D_t)$, mean μ_t, variance σ^2
State variables	
I_t	End-of period inventory for period t, we assume $I_0 = 0$
B_t	End-of period capital for period t
Decision variables	
Q_t	Ordering quantity at the beginning of period t
R_t	whether the retailer orders at period t
S_t	Order up to level at the beginning of period t, and $S_t = I_{t-1} + Q_t$
s_t	Threshold of the inventory level for (s, S) policy

Table 1: Notations adopted in our paper

Solving the multi-level capacitated lot-sizing problem with lead time consideration

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1 Introduction

Material requirements planning and scheduling are strongly related. Solving these two levels in a hierarchical manner can quickly lead to suboptimal or infeasible plans. The reason is that the classical MLCLSP neglects scheduling by assuming a lead time of zero or one. In [1] that issue is solved by developing a model which synchronizes the batches (MLCLSP-Sync). However, most standard MLCLSP benchmark instances turned out to be infeasible using these more detailed problem formulations, i.e., it was not possible to find a feasible solution fitting both levels. Moreover, the model is hard to solve such that for some instances the solution status remained unknown. That is, CPLEX was not able to determine whether the problem is feasible or not within an acceptable time frame. Consequently, the following conclusions derive from these results. It is necessary to solve the MLCLSP and the scheduling part in an integrated model. Otherwise, MCLSLP solutions are likely to be infeasible at the scheduling level. The MLCLSP-Sync has to be extended in order to provide (practical) feasible solutions for every instance. Furthermore, a solution method is needed for the MLCLSP-Sync in order to provide faster and better solutions.

2 Problem description

The MLCLSP-Sync of [1] showed that only a very small number of standard MLCLSP instances is feasible for the case with lead time. Consequently, the model needs to be relaxed in those cases in order to provide a solution feasible in practice. In literature different approaches are common to ensure feasibility. Most prominent are overtime, backorders and subcontracting[1]. The following reasons speak against the usage of the first two.

- Overtime: Weakens the formulation because the capacity limit serves as big-M in several constraints. Overtime allows unlimited capacity and thus requires the formulation of a new big-M which is not as tight as before. Moreover, in practice overtime is not endless but limited.
- Backorders: In a multi-level structure the consequences of backorders on the other stages would have to be modeled. The formulation would get even more complex and thus harder to solve.

Subcontracting means that every missing unit is assumed to be purchased from another company at a price c_i^{sub}. This approach has several advantages. First of all, it is not unrealistic to purchase missing parts in practice. Secondly, subcontracting variable X_{it}^{sub} clearly shows at which stage (item i) and when (period t) a shortage would occur.

2.1 MCLSP-Sync with subcontracting

The MLCLSP-Sync of [1] is adapted as follows. The objective function is extended by multiplying subcontracting cost c_i^{sub} of item i with the subcontracted amount X_{it}^{sub} of item i in period t, see (1). Inventory holding cost h_i occur for item i in period t according to inventory level I_{it}. If there is a changeover T_{ijtm} from item i to item j in period t on machine m setup cost c_{ij} have to be paid. The inventory balance constraints (2) calculate the inventory level of item i in period t based on the inventory level of the previous period I_{it-1}, the production amount X_{it} and subcontracting amount X_{it}^{sub} of item i in period t. This amount is reduced by internal demand, i.e., if an amount X_{jt} of successor item j is produced in period t a_{ij} units of i are needed per unit of j. The inventory level is further reduced by the external demand E_{it}. Constraints (3) are also taken from the MLCLSP-Sync where they are used to determine the production amount $\hat{X}_{ijt}$ of item j starting before predecessor item i is finished in period t. This is only possible if the inventory level I_{it-1} of predecessor item i is sufficiently high or if the required amount of i is subcontracted. For the unchanged constraints (8)-(16) and (18) we refer to [1].

$$\min \sum_{i=1}^{N} \sum_{t=1}^{T} (h_{it} \cdot I_{it} + c_i^{sub} \cdot X_{it}^{sub}) + \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{t=1}^{T} \sum_{m=1}^{M} c_{ij} \cdot T_{ijtm} \tag{1}$$

[1] Another concept which is mathematically identical to subcontracting are lost sales. However, lost sales do not make sense in a multi-level structure because a product cannot be produced if the predecessor is not available.

subject to

$$I_{it} = I_{i(t-1)} + X_{it} + X_{it}^{sub} - \sum_{j \in \Gamma(i)} a_{ij} \cdot X_{jt} - E_{it} \qquad i,t \qquad (2)$$

(8)-(16),(18) in [1]

$$X_{it}^{sub} + I_{i(t-1)} \geq \sum_{j \in \Gamma(i)} a_{ij} \hat{X}_{ijt} \qquad i,t \qquad (3)$$

$$X_{it}^{sub} \geq 0 \qquad i,t \qquad (4)$$

2.2 Subcontracting cost c_i^{sub}

In order to choose subcontracting cost c_i^{sub} appropriately we have set it in relation to the potential production cost, i.e. setup and holding cost. We want to ensure that subcontracting is only used when there is not enough capacity, in other words, own production should always be cheaper. Therefore, we get the following inequality.

$$X_{it} \cdot c_i^{sub} > X_{it} \cdot h_{it} \cdot T + \max_j c_{ji} + \sum_{j \in \Gamma^{-1}(i)} c_j^{sub} \cdot a_{ji} \cdot X_{it} \qquad (5)$$

$$c_i^{sub} > h_{it} \cdot T + \frac{\max_j c_{ji}}{X_{it}} + \sum_{j \in \Gamma^{-1}(i)} c_j^{sub} \cdot a_{ji} \qquad (6)$$

Obviously, the fulfillment of inequality (6) depends on variable X_{it}. Since $X_{it} \geq 0$ can become very small, i.e. close to zero, we can never ensure the validity of the inequality for any c_i^{sub}. This has an influence on our solution methods.

3 Computational tests

For the given problem two solution approaches with roots in multi-objective optimization were tested. First, the presented model was just solved with an MIP solver. This is considered as the weighted-sum approach where several (conflicting) objectives are combined in a single function by using weights to indicate priorities. In our case the weights are the given cost for setups and holding inventory as well as the (artificial) subcontracting cost. We refer to it as MIP-WS. The second approach follows the idea of the ϵ-constraint method and is therefore called MIP-ϵ. The reason is that our primary goal is to find a feasible solution. Therefore, first the MLCLSP-Sync is solved by minimizing subcontracting. Afterwards, subcontracting X_{it}^{sub} is limited by the obtained subcontracting solution and then total cost are minimized. In the optimal case we solve just the MLCLSP-Sync without subcontracting.

Both approaches were tested for a deterministic time limit of 5.76m CPLEX ticks (about 7200 sec. on a computer cluster with Intel Xeon E5-2687W Processors at 3.1

GHz with 256 GB of RAM) for the class1-class6 instances of [2]. For MIP-ϵ we set the time limit for step 1 (minimizing subcontracting) to 2.88m CPLEX ticks. If step 1 stopped before this limit was hit the remaining time was added to the time limit of step 2. In total the deterministic time limit was 5.76m CPLEX ticks as well.

The following observations could be taken from the results. For the small instances of class1 and class2 the results were almost identical, therefore we consider here only class3-6.

- MIP-ϵ found much more solutions without subcontracting than MIP-WS. For classes 3,4,5,6 MIP-ϵ found in 75%, 60%, 87%, 48% of all cases a solution without subcontracting whereas MIP-WS was successful only for 42%, 4%, 9% and 0% of the instances.

- For the complete objective function comprising setup, holding and subcontracting cost MIP-ϵ was better for 71%, 96%, 93% and 100% of all instances (classes 3,4,5,6).

- Although MIP-ϵ neglects setup and holding cost in the first step, at the end it found more often a better solution regarding the sum of these two cost types. It was superior in 70%, 90%, 70% and 58% of all cases (classes 3,4,5,6).

Consequently, extending the MLCLSP-Sync by allowing subcontracting is a helpful relaxation technique. It enables us to find a feasible lot-sizing and scheduling solution easily. Searching for a solution with minimal subcontracting first (MIP-ϵ) turned out to be superior to simply solving the MIP with the weighted sum approach (MIP-WS). Further improvement is expected from reformulations. In [3] several of the classical MLCLSP reformulations were also tested for the MLCLSP-Sync regarding LP bounds and time for first feasible solutions.

References

[1] C. Almeder, D. Klabjan, R. Traxler, and B. Almada-Lobo. Lead time considerations for the multi-level capacitated lot-sizing problem, *European Journal of Operational Research*, 241 (3):727-738 (2015)

[2] H. Tempelmeier, L. Buschkühl. A heuristic for the dynamic multi-level capacitated lotsizing problem with linked lotsizes for general product structures, *OR Spectrum*, 31.2: 385-404 (2009)

[3] A. Malogulko. Tight formulations for the multi-level capacitated lotsizing problem, *Master thesis*, European University Viadrina Frankfurt/Oder (2016)

A computational comparison of formulations for an integrated three-level lot sizing and transportation problem with a distribution structure

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Abstract

We address a three-level lot sizing and transportation problem with a distribution structure (3LSPD). We consider one production plant that produces one type of item over a discrete and finite planning horizon. The items produced are transported to warehouses and then to retailers using direct shipments. Each retailer is linked to a unique warehouse and there are no transfers between warehouses nor between retailers. The objective is to minimize the sum of the fixed production and ordering costs and of the unit variable inventory holding costs. We compare 12 different MIP formulations to solve the problem without production nor transportation capacities. All these formulations are adapted from the MIP formulations proposed for the One-Warehouse Multi-Retailer literature, and most of the formulations proposed are newly introduced in the context of the 3LSPD. We run experiments on both a balanced and an unbalanced network (in the balanced network each warehouse serves the same number of retailers whereas in the unbalanced network 20% of the warehouses serve 80% of the retailers). Our results indicate that the richer formulations are not necessarily the best ones in terms of total CPU time, and that the unbalanced instances are harder to solve.

1 Introduction

Over the last decades, lot sizing problems have attracted the attention of many researchers, mainly because of their numerous applications in production, distribution and inventory management problems. Extensions of the basic lot sizing problem (LSP) are often encountered in the context of supply chain planning. Usually, the customers of a company, which have a certain demand, are located in a different area from the production plant where the items are actually produced and where lot sizing decisions are made. This leads to a transportation problem where the company needs to determine when to deliver the products to its customers so as to minimize the transportation costs. Companies facing these two operational problems often make decisions in sequence, where the output of the lot sizing problem becomes the input of the transportation problem. This leads, however, to solutions that can be far from the optimal solution of an integrated lot sizing and transportation problem.

We address here an integrated and uncapacitated three-level lot sizing and transportation problem with a distribution structure (3LSPD). We consider a general manufacturing company that has one production plant (level one), several warehouses (level two) and multiple retailers (level three) facing a dynamic and known demand for one item over a discrete and finite time horizon. The supply chain considered here has a distribution structure: the warehouses are all linked to the single plant and all retailers are linked to exactly one warehouse. When we consider the demand of a particular retailer, the flow of goods in the supply chain network is hence as follows: an item is produced at the production plant, then sent to the warehouse linked to the retailer for storage and finally sent to the retailer to satisfy its demand. Figure 1 illustrates this flow of goods in a distribution network which consists of one production plant, three warehouses and three retailers linked to each warehouse. The objective of the problem is to determine the optimal timing and flows of goods between the different facilities while minimizing the operational and transportation costs in the whole network (sum of the fixed setup and transportation costs and unit inventory holding costs).

The motivation to work on MIP formulations for the 3LSPD is to extend the works of Solyalı and Süral [1] and Cunha and Melo [2] who compare several MIP formulations for the One-Warehouse Multi-Retailer problem (OWMR).

2 Formulations

The MIP formulations we propose are classified in three families: the classical based formulations, the echelon stock based formulations and the rich formulations. The classical based formulations extend the basic MIP formulation for the ULSP as used by Pochet and Wolsey [4], and the reformulations that have been proposed in the

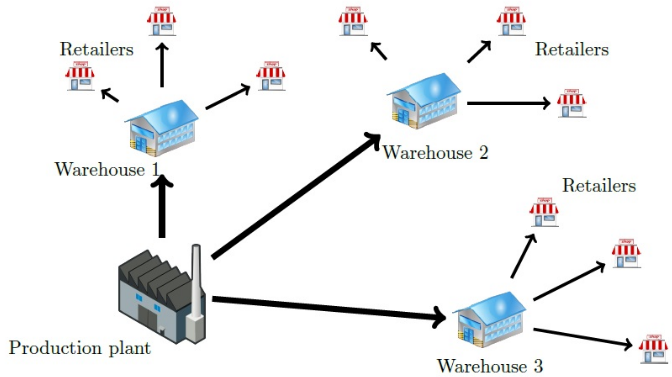

Figure 1: Graphical representation of the problem considered

literature for the single item LSP. Indeed, we observe a single item lot sizing structure for each retailer in the adaptation of this basic formulation. This means that we can use the existing reformulations of the ULSP for each retailer. Such reformulations are however not applicable to the warehouse or plant level, since they face a dependent demand. Therefore, the classical based formulations contain the classical formulation C, the classical network C-N and the classical transportation C-T formulations (where, compared to the C formulation, a network and a transportation reformulation are used at the retailers' level, respectively). Finally, one can also use the polyhedral results for the SI-ULSP to improve the classical formulation C at the retailers' level. We thus incorporate the (l, S, WW) valid inequalities and obtain the C-LS formulation.

Employing the idea of an echelon stock presented in Federgruen and Tzur [3], the 3LSPD can be decomposed into several independent SI-ULSP. To do so, the inventory variables of the classical formulation C are replaced with echelon stock variables representing the total inventory of the one item at all descendents of a particular facility. This leads to the echelon stock formulation ES. Note that with the introduction of the echelon stock variables, the problem has an uncapacitated lot sizing structure with independent demand at each level. This means that we can apply the known reformulation techniques for the ULSP at each level to obtain the ES-N, ES-T and ES-LS formulations, where the network reformulation, the transportation reformulation and the (l, S, WW) inequalities are used, respectively.

The richer formulations contain the network formulation N, the transportation formulation T and the multi-commodity formulation MC. The N formulation comes from the property of extreme flows in a network applied to our problem. The T formulation comes from the interactions between the facilities which are modeled based on the transportation formulation for the SI-ULSP. Finally, the MC formulation is based on the distinction of each retailer-time period pair. These three formulations are richer in the sense that the decision variables used contain more information than the decision variables used in the other formulations. They also have a much higher number of decision variables.

3 Numerical results

In order to assess the strenghts and weaknesses of the different formulations, we conducted computational experiments by adapting the instances used in [1]. In order to contrast our results, we also define two structures for the distribution network represented in Figure 1. In the first structure, we consider a balanced network where each warehouse has the same number of retailers. In the second structure, we consider an unbalanced network where 80% of the retailers are assigned to 20% of the warehouses. We performed our experiments using CPLEX 12.6.1.0 C++ library with a time limit of three hours. We compare all solutions with respect to different indicators: CPU time, number of optimal solutions obtained, cost of the optimal solution/best solution found, number of nodes in the branch-and-cut tree, and integrality and optimality gap. We also take a look at the LP relaxation and the time taken to obtain it.

Our results show that the unbalanced instances are harder to solve and that the MC formulations suits best our problem, both for the balanced and unbalanced instances. The rich formulation N has trouble finding the optimal MIP solution for many instances, despite having the best LP relaxation. The classical based formulations have a poor performance, even if reformulations are used. The echelon stock based formulations, however, benefit from the reformulation techniques used in the formulations ES-N, ES-T and ES-LS to get good performances, but are still worse than the performances of the MC formulation.

References

[1] O. Solyalı and H. Süral, The one-warehouse multi-retailer problem: reformulation, classification, and computational results, Annals of Operations Research, 196, 517-541 (2012)

[2] J. O. Cunha and R. A. Melo, On reformulations for the one-warehouse multi-retailer problem, Annals of Operations Research, 238, 99-122 (2016)

[3] A. Federgruen and M. Tzur, Time-Partitioning Heuristics : Application to One Warehouse, Multiitem, Multiretailer Lot-Sizing Problems, Naval Research Logistics, 46, 463-486 (1999)

[4] Y. Pochet and L. A. Wolsey, Production Planning by Mixed Integer Programming, 207-234. Springer, New York, NY, USA (2006)

Coordinating a two-level supply chain with capacity reservation and subcontracting

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Abstract

We consider a two-level supply chain with one supplier and one retailer that has to meet a non-stationary demand for a single product. We suppose that the supplier has the market power to impose his planning decisions to the retailer. This may induce a large cost for the retailer. We assume that the supplier knows the demand and the retailer's cost parameters. The purpose of our work consists in devising coordination mechanisms for the planning decisions of the actors within the supply chain in order to minimize the retailer's cost. The production plan imposed by the supplier can be viewed as a capacity reservation contract for the retailer. In order to minimize his cost, the retailer can propose to the supplier a contract composed of production and replenishment plans, as well as a side payment. Designing such contracts lead to particular lot-sizing problems with subcontracting aspects. We analyze the complexity of these problems and discuss solving issues for some specific cases.

1 Introduction

We consider a two-level supply chain composed of one supplier and one retailer that has to satisfy a demand for a single product over a planning horizon of T discrete periods. We suppose that the supplier knows the demand at the retailer level. Ordering units at period t induce a unit ordering cost p_t^S (resp. p_t^R) and a fixed ordering cost f_t^S (resp. f_t^R) for the supplier (resp. retailer). Carrying units in the inventory induce a unit holding cost h_t^S and h_t^R for the supplier and the retailer respectively.

We address the problem of coordinating the planning decisions of this supply chain by assuming that the supplier has the market power to impose his optimal production plan to the retailer such that the demands can be satisfied. The optimal production plan of the supplier is determined by minimizing the supplier's total cost where all the units are stored at the retailer's level. Without coordination, the replenishment plan of the retailer is equal to the production plan imposed by the supplier which can induce a large cost for the retailer.

When the supplier has the market power, the problem of coordinating the planning decisions is different from the case where the market power is owned by the retailer [3, 4]. The production plan that is imposed to the retailer can be considered as a capacity that is reserved by the supplier such that the demand can be satisfied. Hence, in our study, we focus on the reservation capacity contract [5]. In this contract, the supplier reserves a certain capacity of units for the retailer. However, when the retailer's ordered units exceeds the capacity, the supplier proposes to the retailer a larger ordering cost denoted by b_t^S for the extra units, i.e the ones exceeding the capacity.

If the actors decide to coordinate their planning decisions, the supplier agrees to order more units at a period t than his optimal ordering if the retailer takes over the ordering cost b_t^S for the extra units. The coordination scheme is the following: the supplier determines his optimal production plan x_{opt}^S of cost C_{opt}^S which represents the reservation capacity. Then, he proposes to the retailer a reservation capacity contract (x_{opt}^S, b^S). Finally, in order to decrease his cost, the retailer can propose to the supplier a contract composed of a replenishment plan and a possible side payment to ensure that the supplier's cost will not increase.

In our study, we propose different contracts by assuming that the optimal production plan of the supplier is modified or not. The problem of designing these contracts leads to particular lot-sizing problems for which we propose a complexity analysis.

2 Contract with stock transfer at the supplier level

We consider the case where the retailer does not modify the optimal production plan of the supplier. However, in order to decrease his cost, he proposes to cover the supplier's holding cost of the units that are stored at the supplier level. In that case, the retailer proposes to the supplier a contract (x^R, z) where x^R is a replenishment plan and z represents a side payment on the supplier's holding cost.

The problem of designing the contract (x^R, z) corresponds to a two-level lot-sizing problem where the supplier's production plan is fixed to his optimal production plan. At first, we show that the retailer can decrease his cost by a factor arbitrarily large by proposing a contract (x^R, z). Then, we propose a dynamic programming algorithm that solves the problem of designing the contract (x^R, z) in $\mathcal{O}(T^2 \log T)$.

3 Contract with subcontracting

The retailer can propose another production plan to the supplier from the reservation capacity contract. The assumption that the supplier does not carry units in his inventory still hold.

In this context, determining a replenishment plan in order to minimize the retailer's total cost is known as a capacitated lot-sizing problem with subcontracting in the literature, denoted by CLS-S. The CLS-S problem is defined as a lot-sizing problem where the production capacity is limited, but it is possible to order more than the capacity by subcontracting at a higher cost. The CLS-S problem can be solved in polynomial time by dynamic programming algorithms when the production capacity is stationary [1, 2, 6].

Ordering more units than the capacity can induce additional fixed ordering costs to the supplier that he did not have to pay in its optimal solution. We propose to study the case where the retailer pay part of the supplier's fixed ordering cost or not.

3.1 Side payment on the supplier's fixed ordering cost

We consider the problem of designing a contract $(\overline{x}^R, z)$ where the retailer pays the additional fixed ordering cost of the supplier. $\overline{x}^R$ is a replenishment plan and z is a side payment on the unit ordering cost b^S and the additional fixed ordering cost of the supplier. The supplier's cost associated to this contract is at most equal to C^S_{opt}.

Let R_t be the capacity reserved by the supplier at period t. The quantity of units ordered at period t which does not exceed R_t is given by x_t. The one that exceed R_t is given by u_t. The setup variable y_t (resp. z_t) is equal to 1 if the setup is (resp. is not) initially in the optimal production plan. The mathematical formulation of this problem is given by:

$$\min \quad \sum_{t \in \mathcal{T}} f^R_t(y_t + z_t) + p^R_t(x_t + u_t) + h^R_t s^R_t + b^S_t u_t + f^S_t z_t$$

$$\text{s.t.} \quad s^R_{t-1} + x_t + u_t = d_t + s^R_t \qquad \forall t \in \mathcal{T}, \tag{1}$$

$$x_t \leq R_t y_t \qquad \forall t \in \mathcal{T}, \tag{2}$$

$$u_t \leq (d_{tT} - R_t)(y_t + z_t) \qquad \forall t \in \mathcal{T}, \tag{3}$$

$$x_t, s^R_t, u_t \geq 0, y_t, z_t \in \{0, 1\} \qquad \forall t \in \mathcal{T}$$

Constraint (1) represent the inventory balance constraint. Constraint (2) ensure that the quantity of units x_t does not exceed R_t. Constraint (3) ensure that the retailer will only pay the additional fixed ordering cost of the supplier.

We show that the supplier's and the retailer's gain associated to the contract $(\overline{x}^R, z)$ can be arbitrarily large.

3.2 No side payment on the supplier's fixed ordering cost

We suppose that the retailer does not pay the supplier's additional fixed ordering cost. He proposes to the supplier a contract $(\tilde{x}^R, z)$ where $\tilde{x}^R$ is a replenishment plan and z is a side payment on the unit ordering cost b^S. In that case, the retailer has to ensure that the contract does not increase the supplier's cost.

Similarly to the problem of designing a contract $(\overline{x}^R, z)$, the supplier's and the retailer's gain associated to the contract $(\tilde{x}^R, z)$ can be arbitrarily large. Moreover, we show that the problem of designing the contract $(\tilde{x}^R, z)$ is NP-hard through a reduction from the knapsack problem.

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References

[1] A. Atamtürk and D. S. Hochbaum, Capacity Acquisition, Subcontracting, and Lot Sizing, Management Science, 47(8), 1081-1100 (2001)

[2] F. Chu and C. Chu, Polynomial Algorithms for Single-Item Lot-Sizing Models With Bounded Inventory and Backlogging or Outsourcing, IEEE Transactions on Automation Science and Engineering, 4(2), 233-251 (2007)

[3] Z. Mobini, W. van den Heuvel, A. Wagelmans, Designing multi-period supply contracts in a two-echelon supply chain with asymmetric information, Econometric Institute Research Papers EI2014-28, Erasmus University Rotterdam, Erasmus School of Economics (ESE), Econometric Institute (2014)

[4] S-L Phouratsamay, S. Kedad-Sidhoum and F. Pascual, Coordination of a two-level supply chain with contracts under complete or asymmetric information, http://www.optimization-online.org/DB_HTML/2017/07/6123.html (2017)

[5] D. A. Serel, M. Dada and H. Moskowitz, Sourcing decisions with capacity reservation contracts, European Journal of Operational Research, 131(3), 635-648 (2001)

[6] M. Zhang, Capacitated lot-sizing problem with outsourcing, Operations Research Letters, 43(5), 479-483 (2015)

A construction heuristic for the Supply Vehicle Routing and Lot Sizing Problem

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1 Introduction

The advantages of integrating production and distribution planning are well known and have been discussed intensively in research. In these so-called production routing problems (PRP) lot sizing is combined with vehicle routing to model the distribution part [1]. Due to increased levels of integration of supply chain partners and especially for assembly-oriented manufacturers, a similar problem occurs on the supply side, i.e., in order to feed the production system with raw materials a wisely chosen logistic system is necessary to ensure a timely and cost efficient supply of raw materials avoiding unnecessary high stocks as well as raw material shortages.

Kuhn and Liske [5] were the first researchers tackling the problem of combining a vehicle routing problem (VRP) on the supply side with an economic lot scheduling problem (ELSP) on the production side. Hein and Almeder [3] extend their approach to a more realistic production setting facing dynamic demand and capacity restrictions. The authors show that an integration of supply and production planning allows for substantial cost savings compared to a classical sequential planning approach particularly if raw material storage is relatively costly compared to the storage of final products.

However, this type of problem is challenging since it integrates two complex planning problems, namely inventory routing and capacitated lot sizing. Thus, an efficient algorithm is needed to be able to solve large instances which does not yet exist. For the counterpart, the PRP, various heuristic algorithms have been developed [1]. However, so far no constructive heuristic has been proposed though the benefits of such heuristics are manifold, e.g., the computation time is extremely low, no commercial solver is needed, it can be easily integrated into existing software, and it is usually better understood and accepted by practitioners. The aim of this work is to provide a construction heuristic for the supply vehicle routing and lot sizing problem (CLSVRP) utilizing common-sense priority rules. After that, these rather intuitive priority rules are replaced by more advanced rules that have been automatically generated by means of genetic programming [4].

2 Problem description

Consider a single-stage production system in which a set of end products are manufactured for which predefined amounts of raw materials are needed. The manufacturer has to solve a standard lot sizing problem (without setup times) in order to meet deterministic but time-dependent demand. Raw materials are withdrawn from the raw material inventories and transformed into final goods. There is a general relation between raw materials and end products, i.e., the same raw material might be used for several end products and an end product is usually made out of several different raw materials. The production amounts per period are limited by a finite capacity. In order to replenish raw material inventories the manufacturer has to collect those raw materials using a homogeneous fleet of vehicles with limited capacities at different suppliers which are geographically dispersed in close proximity to the production facility. We assume that a tour of a vehicle starts and ends in the same time period such that the raw materials collected by those vehicles are available for production in the following time period. The overall objective is to minimize total cost consisting of setup cost, holding cost for end products, holding cost for raw materials, fixed and variable transportation cost.

3 Model outline

A mathematical formulation of the CLSVRP has been introduced by [3]. The integrated model contains a production part represented by a capacitated lot sizing problem (CLSP), and a routing part modeled similar to an inventory routing problem (IRP). In the CLSP production amounts are limited by the availability of raw materials. The IRP used to model the routing part is, in fact, a routing problem where materials are collected from different locations (assuming infinite supply) and stored at the depot (= production plant) waiting to be used in the production process. Both parts are linked by the inventory balance equation of the raw materials.

4 Construction heuristic

In this section we propose a simple construction heuristic for the CLSVRP which is built upon the Dixon-Silver (DS) heuristic, a construction heuristic originally designed for solving the CLSP [2]. The DS algorithm follows an iterative procedure in which a production schedule is generated period-wise. Crucial decision criteria for the lot formation are (i) the sequence in which products are planned, and (ii) the boolean decision on lot extension. For both decisions, DS uses a priority index u^{DS} which is defined as:

$$u_{i\tau}^{DS} = \Big(\frac{SC_{i\tau} + HC_{i\tau}^{T}}{T_i} - \frac{SC_{i\tau} + HC_{i\tau}^{T+1}}{T_i + 1}\Big)/k_i d_{i,\tau+T_i} \tag{1}$$

Let $SC_{i\tau}$ denote the setup cost of product i in the current planning period τ, let $HC_{i\tau}^{T}$ and $HC_{i\tau}^{T+1}$ denote the total holding cost incurred with the lot of product i setup in τ covering the demand of T and $T+1$ periods, respectively, and let $k_i d_{i,\tau+T_i}$ indicate the additional capacity requirements needed to include the demand of period $\tau + T_i$ in the current lot. Finally, u^{DS} expresses the difference in the average total costs per period per additional capacity unit required. For criterion (i), items are sorted according to non-increasing u^{DS}, for criterion (ii), a lot extension is confirmed if cost savings can be achieved, i.e. $u^{DS} \geq 0$.

In order to simultaneously build a production plan and a supply schedule, we adapt u^{DS} by including the approximate supply cost in the definition of the priority index ensuring that supply cost are already regarded during the lot sizing step. The cost of raw material supply arising with the production of end product i, denoted as inventory routing cost IRC_i, are determined by a cheapest insertion heuristic. Thus, the modified priority index u^{DS-VRP} for solving the CLSVRP is defined as:

$$u_{i\tau}^{DS-VRP} = \Big(\frac{SC_{i\tau} + HC_{i\tau}^{T} + IRC_{i\tau}^{T}}{T_i} - \frac{SC_{i} + HC_{i\tau}^{T+1} + IRC_{i\tau}^{T+1}}{T_i + 1}\Big)/k_i d_{i,\tau+T_i} \tag{2}$$

The computation of $IRC_{i\tau}$ comprises the following steps:

- *Compute raw material requirements:* for each raw material $m \in \mathcal{S}_i$ with $\mathcal{S}_i$ being the set of raw materials of end product i, compute the amounts needed to produce $d_{i,\tau+T_i}$ units of i

- *Determine supply cost of raw materials:* for each $m \in \mathcal{S}_i$ check the supply cost in lexicographical order:

  - *Determine vehicle routing cost:* seek for the cheapest insertion of the corresponding supplier in any existing tour of period $t = \{\tau, \tau - 1, ..., 1\}$; insertion cost in t are denoted as VRC_{mt}

  – *Find cheapest supply period:* for each t compute the supply cost consisting of routing cost VRC_{mt} and holding cost of raw materials HC_{mt}^{raw}, find the cheapest supply period, i.e. set $IRC_{m\tau}^{raw} = \min_t \{VRC_{mt} + HC_{mt}^{raw}\}$

- *Calculate total supply cost:* $IRC_{i\tau} = \sum_{m \in S_i} IRC_{m\tau}^{raw}$

The heuristic proceeds similar to the original DS algorithm. Each time a change of the lots in the production plan is confirmed, the routing schedule is updated accordingly requiring a recalculation of u^{DS-VRP}.

A genetic programming enhanced construction heuristic Within the structure of the extended DS algorithm, we identified three crucial decisions which relate to: (i) the sequence of end products, (ii) lot extension, and (iii) the sequence of raw materials in which they are checked for computing supply cost. Our aim is to replace these rather intuitive decision rules by more advanced rules automatically generated through genetic programming (GP). Genetic programming belongs to the class of evolutionary algorithms and is able to automatically construct mathematical expressions typically represented as parse trees suitable for solving a given problem [4].

References

[1] Y. Adulyasak, J.-F. Cordeau, R. Jans. The production routing problem: A review of formulations and solution algorithms. Computers & Operations Research, 55, 141-152 (2015)

[2] P.S. Dixon and E.A. Silver. A heuristic solution procedure for the multi-item, single-level, limited capacity, lot-sizing problem. Journal of Operations Management, 2, 23-39 (1981)

[3] F. Hein and C. Almeder. Quantitative insights into the integrated supply vehicle routing and production planning problem. International Journal of Production Economics, 177, 66-76 (2016)

[4] J.R. Koza. Genetic Programming: On the Programming of Computers by Means of Natural Selection, MIT Press, Cambridge, Massachusetts (1992)

[5] H. Kuhn and T. Liske. Simultaneous supply and production planning. International Journal of Production Research, 49, 3795-3813 (2011)

The Inventory Routing Problem in Rectangular Warehouses

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Abstract

In warehouses, storage replenishment involves the transportation of items to the capacitated item slots in forward storage area from reserve storage. These items are later picked from these slots if necessary. While picking constitutes the majority of operating costs, replenishment might be as costly if pick lists consist of only a few lines. We consider the storage replenishment problem in a rectangular warehouse, where replenishment and picking are carried out in successive waves with time limits. The problem is analogous to the inventory routing due to the inherent trade-off between labor and travel costs. We show that the problem is NP-hard. We use a heuristic inspired from the literature. We analyze the effect of storage policies and demand patterns and compare our approach to those in practice.

1 Introduction

Breakdown of warehouse operating costs reveals that making necessary items available in pick area and picking of these items for satisfying customer orders cover 55% of the total costs. Although the order picking problem (OPP) is a well-studied problem, to the best of our knowledge, its relation with replenishment activities is not considered in the literature. In this study, we consider a coordinated approach, where the replenishment and routing decisions are made in an integrated manner by taking into account the dependence between the replenishment and pick cycles. In doing so, we aim to complete the replenishment operations within pre-specified time limits before picking operations, hence avoiding overtime, while minimizing the transportation costs by making use of economies of scale during replenishment.

Order picking operations require items to be available in the storage area prior to picking. Upon receipt, items are put away into the reserve storage area, where they are stored in bulk amounts. According to the picking schedule, items are broken down and replenished into the forward storage area, where storage is in smaller quantities. The motivation is to sacrifice space efficiency and provide better accessibility of items for more efficient picking. Hence, the availability of items in the forward storage area is ensured by replenishing the needed items from the storage area.

In general, replenishment and picking activities are performed in sequence in a cyclic manner. Each of the replenishment and pick cycles is called a “wave.” When planning for replenishment, these waves are treated independently, that is, the decisions of which items to replenish and how much are made based only on the upcoming pick wave. In this case, routing decisions for replenishment in the forward storage area are made mostly identical to those of order picking. Two issues might arise: (i) The replenishment wave might exceed the time limit, resulting in the need for overtime, and (ii) Treating each wave independently might result in excessive transportation.

To the best of our knowledge, the case of replenishment of items in the forward storage area is not considered in conjunction with order picking cycles, which is a gap that is aimed to be filled in this study.

2 Problem Definition and Complexity

The replenishment and picking tasks are performed in a cyclic manner. We assume equal cycle lengths and the customer orders to be picked in each pick cycle are known.

We define the *storage replenishment problem* (SRP), which aims to make decisions on (i) when and how much to replenish each item in the pick area from reserve storage to guarantee availability and (ii) the routing of the replenishment carrier in each period. In the SRP, we assume a single uncapacitated replenishment carrier available. A set of “periods” (waves), their pre-specified time lengths of waves, and arrivals of each item at the reserve storage in each day are known. The warehouse layout, item locations with corresponding storage capacities, inventories of items in the reserve and forward storage areas, and the amount of each item to be picked in each wave are given. Demand patterns (uniform or skewed) and storage policies (random or turnover-based) are specified and known. The objective in the SRP is to minimize the total replenishment travel time. Under these settings, the SRP is similar to the Inventory Routing Problem (IRP). The (dis)similar features that (de)construct the two problems can be specified as follows. There exists one-to-one correspondence between “supliers” and “retailers” in the IRP and “reserve storage area” and “item locations". "Demand time and amounts" in the IRP match with "pick lists" of the SRP while "load capacity" of IRP can represent "wave time limit" of the SRP. A "single item and multiple retailers" structure of the IRP can be seen as a "multiple items, each of which demanded by one retailer". Although "holding cost" of the IRP is an important trade-off component, the SRP does not incorporate such a cost item. However, "availability" may point out an imputed cost in the SRP. Another distinction between two problems is about routing decisions because the SPR has a special structure that makes routing "easier" than that of the IRP.

3 An A Priori Route-Based Heuristic

The NP-completeness of the SRP suggests that as the instance size grows, the solution time increases. To overcome this burden, we propose an a priori route-based heuristic so that once the items to visit are fixed, the routing problem is easily solvable. Solving the routing problem faster differentiates our work from that by [4].

In the first step of the heuristic, we solve the OPP corresponding to all the items that will be picked throughout the planning horizon. To do so, we may use the exact approach by [3] or the heuristics proposed by [2]. Once the a priori route is determined, we fix the sequence of items to be visited in each replenishment trip in order to simplify the routing decisions in the next step. The first two steps of the heuristic determine the route of replenishment, given which items will be replenished. This leaves the decision of which items to replenish (note that how much to replenish is not a part of the decisions). For this end, we extend the strong formulation of the reduced model for the IRP, for which the idea was applied by [4].

The model uses the precedence sets for each item determined by the previous step and the w_{ikt} values for each item as additional parameters, and decides on whether an arc on the network will be used or not, subject to inventory balance, network flow balance, wave time limit, and routing precedence constraints. As the last step, given which items will be replenished in each wave, the resulting routes are improved using the Ratliff and Rosenthal algorithm for these waves. Note that w_{ikt} denotes whether item i is replenished in wave t, after the last replenishment was made in wave k.

4 Computational Experiments

The objectives of computational experiments are as follows: compare different a priori routing approaches among each other; measure the effect of instance size, demand structure, and storage policy on the efficiency of replenishment; and analyze the extent of the improvement of the replenishment schemes over those in practice.

We consider demand as uniform, where the probability that any item will appear on a pick list on a given day is identical; and skewed, where items have more likelihood to appear on the pick list. For the latter, we use the Pareto distribution with 20-80 skewness, i.e., 20% of the items receive 80% of the total demand. We analyze two storage policies: items are stored randomly; and turnover-based storage where items with more demand are stored "in good locations." 75 instances are based on [1]-[4].

The S-shape heuristic is closer to the optimal route when the density of the items in the warehouse is high. This is reflected in the results by the decreasing gap of it from 15 to 75 items. The largest gap heuristic, which performs better when item density is lower, displays an increasing gap level with increasing number of items.

For demand and storage policy effect, the results are best when demand is skewed and turnover-based storage is used. If demand is skewed, applying a turnover-based storage has an average travel time savings of 6% on average, underlining the importance of using a storage policy in line with demand skewness. When demand is skewed and turnover-based storage is applied, an average of 4% less time is observed compared to uniform demand and random storage. When storage policy is in line with demand skewness, it yields an advantage over the case when demand is uniform.

We show the percent improvement of the heuristics under different a priori route schemes from the method where each wave is treated independently as being treated in practice. The improvement ranges from 15-31% for optimal, from 9-28% for S-shape, and 9-23% for largest gap a priori routes.

5 Conclusion and Further Research Directions

New research directions involve the extension of the work to multiple capacitated replenishment pickers and warehouses with different layouts.

References

[1] C. Archetti, L. Bertazzi, G. Laporte, and M.G. Speranza. "A branch-and-cut algorithm for a vendor-managed inventory-routing problem." *Transportation Science* 41(3), 382–391 (2007).

[2] R. W. Hall. "Distance approximations for routing manual pickers in a warehouse." *IIE Transactions* 25(4), 76–87 (1993).

[3] H. D. Ratliff and A. S. Rosenthal. "Order picking in a rectangular warehouse: A solvable case of the Traveling Salesman Problem." *Operations Research* 31(3), 507–521 (1983).

[4] O. Solyalı and H. Süral . "A branch-and-cut algorithm using a strong formulation and an a priori tour-based heuristic for an inventory-routing problem." *Transportation Science* 45(3), 335–345 (2011).

A review of methods for modeling multiple transportation modes in inventory management models

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1 Introduction

Companies buy transportation services from carriers offering various transportation modes, which are characterized by vehicle or container type and capacity, as well as a specific cost function and a lead time. Within physical modes (road, rail, sea, air), several options exist depending on the shipment size, the type of service impacting the transportation lead time (emergency or regular) or the type of cargo, for example frozen, oversized, bulk and general carriers. Typical modes used for different shipment sizes include parcel, Less than Truck Load (LTL) and Full Truck Load (FTL). The parcel carriers are usually used for small shipments, LTL for intermediate and FTL for large shipments. The size of the trucks or containers varies, and depending on the packaging option (for example the type of pallets), the maximum load capacity is different. For the FTL mode, a fixed fee is charged per container or vehicle regardless the filling rate. A realistic cost structure for LTL modes is a piecewise linear, all-unit discount function with a minimum shipment charge to discourage extremely small shipments. Over-declaring is a common practice for LTL shipments to obtain lower price corresponding to the next rate breakpoint.

Hence, the complexity of modeling and choosing the optimal transportation mode has increased due to more transportation options and pricing schedules offered by carriers. Despite the fact that the share of transportation in logistics costs is high, the majority of the existing inventory management models neglect or simplify transportation costs, often assuming that only one transportation option is available. In this presentation, we review and classify the methods for modeling multiple modes and identify new areas for research (see [4] for more details).

2 Transportation costs and mode usage

The following ways of modeling transportation costs in inventory models have been found in the inventory management literature: Constant unit cost, fixed charge function, FTL function, LTL function, approximation function, carload discount schedule, combined replenishment function. According to [6], the transportation expense is often omitted or assumed fixed when the buyer decides replenishment quantities, and this inaccuracy can easily overwhelm any savings related to good inventory management. Transportation costs are also often oversimplified by disregarding discount schedules, transportation capacity limits, as well as availability of multiple modes. In practice, shippers may choose among different transportation alternatives and switch from one to another as needed. In the literature on supplier selection, order split has been widely studied [2]. However, the number of studies that consider multiple transportation modes is limited and can be divided into the following groups based on their assumptions on transportation mode usage during the planning horizon:

1. Multiple transportation modes are available, but only one mode can be used during the whole planning horizon [7],

2. Multiple transportation modes are available, but only one mode can be used at each time period, however, this mode can be different for each period [3],

3. Multiple transportation modes can be used simultaneously, i.e. combined, and each mode can deliver a fraction of the order in each period [1].

Combining transportation modes corresponds to order split in supplier selection decisions, when the total order quantity is split among several suppliers. The reasons for order split can be a total cost reduction, reduction of dependency on a single supplier, transportation lead time reduction or if the total demand is larger than the supply capacity of a single supplier, etc. Order split, also termed multiple sourcing, is often used in stochastic demand settings to reduce the risk of stock-out situations or the costs of safety stocks, typically combining regular and emergency shipments. The benefits of mode combinations can be cost and emission savings, risk mitigation in case of disruptions, access to extra capacity in addition to internal fleet.

3 Methods for modeling multiple modes

In situation where multiple transportation modes are available, the transportation costs can be modeled in the following ways.

1. Combining all modes into a single cost function, where each quantity corresponds to a predefined mode. When combining different modes into a single

cost function, the quantity shipped by each mode is not modeled explicitly. The decision variable reflects the total shipping quantity, which corresponds implicitly to the pre-defined type of mode that is used. Two approaches for the first method are identified: Pre-processing and use of a "car-load" discount schedule:

- The pre-processing approach includes combining transportation cost functions for all modes and creating a new general cost function, where each quantity is associated with the cost of the mode with the lowest cost for this quantity, see for example [3]. It is assumed that only one transportation mode can be used for each shipment quantity. This approach can be used to create a general cost function combining different discount schedules, and is only valid if it is easy to determine which rate is superior for each mode. The drawbacks of this approach include worse solutions compared to other methods (as modal split is not allowed), the need for a pre-proceeding procedure to create a general cost function, a very large number of break points and a-priori determination of the maximum shipment quantity, without the possibility to define constraints on a specific mode.
- A "carload" discount schedule can only be used for modeling two modes, FTL and LTL, with the same cargo limits in the same cost function. These two modes are treated as one mode, where a switch to FTL mode is based on the shipped or over-declared quantity, as in [8]. Freight rates from the carriers can be directly used in this approach, and it is easy to model without pre-processing. However, only two modes (FTL and LTL with single quantity interval) with identical maximum capacity can be modeled using a "carload" discount schedule approach.

2. The cost function of each mode is modeled explicitly, i.e. the decision variables reflect the quantity shipped by each mode. The main benefit is that lower costs can be obtained compared to the first approaches. Using this approach, the freight rates from carriers can be directly used in the model, and it is easier to apply restrictions per mode or allowed combinations. However, the number of decision variables increases when the number of modes increases. Several authors have considered that several FTL or multiple set-up modes are available and that it is possible to combine them for the same shipment (see for example [5] and [1]). However, in these studies, only multiple FTL modes are modeled, while LTL modes are not considered. In addition, some of the reviewed models assume bulk shipments and that the modes always fully utilize the capacity. Such assumption is not always realistic in practice for other types of shipments.

4 Conclusions

Various methods for modeling transportation costs and multiple modes have been proposed in the inventory management literature. Based on the performed review, we propose several directions for future research that will be detailed in the presentation:

- Development of methods to increase the computational efficiency for solving models with realistic discounts and a large number of multiple modes with different capacities,
- Investigation of the conditions for using various methods for mode selection, achieving savings from changing or combining modes, parameter analysis and managerial recommendations.

References

[1] Absi, N., Dauzère-Pérès, S., Kedad-Sidhoum, S., Penz, B., Rapine, C., 2013. Lot sizing with carbon emission constraints. European Journal of Operational Research 227 (1), 55–61.

[2] Aissaoui, N., Haouari, M., Hassini, E., 2007. Supplier selection and order lot sizing modeling: A review. Computers & operations research 34 (12), 3516–3540.

[3] Diaby, M., Martel, A., 1993. Dynamic lot sizing for multi-echelon distribution systems with purchasing and transportation price discounts. Operations Research 41 (1), 48–59

[4] Engebrethsen, E., Dauzère-Pérès, S, 2017. Transportation modes in inventory models: a literature review. Submitted.

[5] Jaruphongsa, W., ÇEtinkaya, S., Lee, C.-Y., 2007. Outbound shipment mode considerations for integrated inventory and delivery lot-sizing decisions. Operations Research Letters 35 (6), 813–822.

[6] Ke, G. Y., Bookbinder, J. H., Kilgour, D. M., 2014. Coordination of transportation and quantity discount decisions, with coalition formation. International Journal of Production Research 52 (17), 5115–5130.

[7] Kutanoglu, E., Lohiya, D., 2008. Integrated inventory and transportation mode selection: A service parts logistics system. Transportation Research Part E: Logistics and Transportation Review 44 (5), 665–683.

[8] Li, C.-L., Hsu, V. N., Xiao, W.-Q., 2004. Dynamic lot sizing with batch ordering and truckload discounts. Operations Research 52 (4), 639–654.

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