

**Local Organising Committee:**

Bernardo Almada-Lobo, Faculty of Engineering, University of Porto, Portugal  
almada.lobos@fe.up.pt

Pedro Amorim, Faculty of Engineering, University of Porto, Portugal  
amorim.pedro@fe.up.pt

Luis Guimarães, Faculty of Engineering, University of Porto, Portugal  
guimaraes.luis@fe.up.pt

Gonçalo Figueira, Faculty of Engineering, University of Porto, Portugal  
goncalo.figueira@fe.up.pt

**Steering Committee:**

Nabil Absi (France)

Christian Almeder (Germany)

Stéphane Dauzère-Pérès (France)

Meltem Denizel (Turkey)

Hark-Chin Hwang (South Korea)

Raf Jans (Canada)

Safia Kedad-Sidhoum (France)

Horst Tempelmeier (Germany)

Wilco van den Heuvel (Netherlands)

Mathieu Van Vyve (Belgium)

Laurence A. Wolsey (Belgium)

**Organised by:**

INESC TEC

Faculty of Engineering, University of Porto

## Welcome

Dear Friends,

Welcome to the IWLS 2014, the 5th edition of the International Workshop on Lot-Sizing. We are honored to host this meeting in Porto, a city that just won the European Best Destination award for the year 2014. We have three intense days ahead of us, but we hope this edition will match the high scientific standards and the relaxed and pleasant atmosphere that characterized the previous meetings.

This workshop covers recent advances in lot-sizing and in this year's program we have dedicated sessions to game theory, inventory/production routing and sustainability extensions to the traditional lot sizing approaches. Besides these topics we will also discuss advances in algorithms and heuristics, new models and the incorporation of uncertainty. It is the first year in which IWLS will be integrated as one of the activities of our Euro Working-Group on Lot-Sizing (EWG LOT). EWG LOT aims to advance the development and application of Operations Research methods, techniques and tools to the field of lot-sizing. During our workshop we will have a Business Meeting to discuss the action plan of the EWG LOT in detail.

In this 5th edition of the International Workshop on Lot-Sizing we are organizing a roundtable session - "Future Perspectives on Lot-Sizing", where forthcoming challenges and opportunities will be discussed. We believe that this may constitute a privileged opportunity to think about our future research avenues in this field. No IWLS would be complete without the social networking opportunities that will take advantage of the wonderful touristic attractions and local cuisine in the city of Porto (not to mention Port wine of course!). We have also organized a social program that includes a welcome drink and dinner, a tram tour followed by a dinner downtown and, finally, a visit to Porto Cellars and riverside dinner.

With more than 50 participants from over 10 different countries, we hope that the program will promote interesting discussions, paving the way for future research in lot-sizing. We wish you a fruitful and memorable stay in Porto.

Best wishes,  
Bernardo Almada-Lobo  
Pedro Amorim  
Luis Guimarães  
Gonçalo Figueira

## Program Schedule

	26 of August	27 of August	28 of August	29 of August
9:30 - 10:00		Opening Session	Inventory / Production Routing	Complexity / Approximability
10:00 - 10:30 10:30 - 11:00		Game Theory		
11:00 - 11:30		Coffee Break	Coffee Break	Coffee Break
11:30 - 12:00 12:00 - 12:30 12:30 - 13:00		Lot Sizing and Extensions	Algorithms / Heuristics	Sustainability
13:00 - 13:30 13:30 - 14:00 14:00 - 14:30		Lunch	Lunch	Closing Session
14:30 - 15:00 15:00 - 15:30 15:30 - 16:00		Stochastic Lot Sizing	Models	
16:00 - 16:30		Coffee Break	Coffee Break	
16:30 - 17:00 17:00 - 17:30		Future Perspectives	Business Meeting	
17:30 - 18:00 18:00 - 18:30 18:30 - 19:00				
19:00 - 19:30 19:30 - 20:00 20:00 - 20:30 20:30 - 21:00 21:00 - 21:30 21:30 - 22:00	Welcome Drink and Dinner	Tram Tour + Downtown Dinner	Porto Cellars + Riverside Dinner	

## Tuesday, 26 August

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### Welcome Drink and Dinner

**19:00 - 22:00**

The Welcome Drink and Dinner will be held in the restaurant Casa Agrícola that is located next to the conference venue at Rua Bom Sucesso 241, 4150 - 150 Porto.

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## Wednesday, 27 August

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<b>09:30 - 10:00</b>	<b>Opening Session</b>
<b>10:00 - 11:00</b>	<b>Game Theory</b> 1. Benders Decomposition applied to cooperative lot-sizing, <i>Andreas Elias and Alf Kimms</i> 2. Competitive uncapacitated lot-sizing game, <i>Margarida Carvalho, Claudio Telha and Mathieu Van Vyve</i>
<b>11:00 - 11:30</b>	<b>Coffee Break</b>
<b>11:30 - 13:00</b>	<b>Lot Sizing and Extensions</b> 1. Simultaneous lotsizing and scheduling with scarce setup resources and multiple setups per period, <i>Karina Copil and Horst Tempelmeier</i> 2. Integration of Lot-Sizing and Scheduling Decisions, <i>Gómez Urrutia, Edwin David, Riad Aggoune and Stéphane Dauzère-Pérès</i> 3. An integrated lot sizing and vehicle routing model for fast deteriorating products, <i>Tom Vogel, Christian Almeder and Pamela C. Nolz</i>
<b>13:00 - 14:30</b>	<b>Lunch</b>
<b>14:30 - 16:00</b>	<b>Stochastic Lot Sizing</b> 1. On robust vs. flexible dynamic lot sizing subject to capacity constraints and random yield, <i>Stefan Helber, Florian Sahling and Katja Schimmelpfeng</i> 2. Extended formulations for static-dynamic uncertainty strategy under various service level measures, <i>Huseyin Tunc, Onur A. Kilic, and S. Armagan Tarim</i> 3. A joint chance-constraint programming approach for a stochastic lot-sizing problem, <i>Céline Gicquel, Jianqiang Cheng and Abdel Lisser</i>
<b>16:00 - 16:30</b>	<b>Coffee Break</b>
<b>16:30 - 17:30</b>	<b>Future Perspectives</b> A roundtable on the Futures perspectives on Lot-Sizing, where the forthcoming challenges and opportunities will be discussed, <i>Bernardo Almada-Lobo, Horst Tempelmeier, Stéphane Dauzère-Pérès and Albert Wagelmans</i>
<b>19:00 - 22:00</b>	<b>Tram Tour + Downtown Dinner</b> Downtown Dinner at restaurant Clérigos that is located at Rua das Carmelitas 151, 4050-367 Porto.

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## Thursday, 28 August

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	<b>Inventory / Production Routing</b>
<b>09:30 - 11:00</b>	<ol style="list-style-type: none"><li>1. A maritime inventory routing problem with stochastic travelling times, <i>Agostinho Agra, Marielle Christiansen, Alexandrino Delgado and Lars Magnus Hvattum</i></li><li>2. An a priori tour-based three phase heuristic for the production-routing problem, <i>Oguz Solyali, Haldun Sural, Jean-François Cordeau and Raf Jans</i></li><li>3. New cutting planes for inventory routing, <i>Pasquale Avella, Maurizio Boccia and Laurence Wolsey</i></li></ol>
<b>11:00 - 11:30</b>	<b>Coffee Break</b>
	<b>Algorithms / Heuristics</b>
<b>11:30 - 13:00</b>	<ol style="list-style-type: none"><li>1. Worst case analysis of relax-and-fix heuristics for lot-sizing problems, <i>Nabil Absi and Wilco van den Heuvel</i></li><li>2. Two neighborhood search approaches for the dynamic capacitated lotsizing problem with scarce setup resources, <i>Sascha Duerkop, Dirk Briskorn and Horst Tempelmeier</i></li><li>3. Towards a Universal Solver for Lot Sizing Problems, <i>Rui Rei and João Pedro Pedroso</i></li></ol>
<b>13:00 - 14:30</b>	<b>Lunch</b>
	<b>Models</b>
<b>14:30 - 16:00</b>	<ol style="list-style-type: none"><li>1. Capacity estimation based on clearing functions for the capacitated lot-sizing problem, <i>Frank Herrmann, Frederick Lange and Christian Almeder</i></li><li>2. The Dynamic Lot-sizing Problem with Convex Economic Production Costs and Setups, <i>Ramez Kian, Ulku Gurler and Emre Guberik</i></li><li>3. The capacitated lot sizing problem with setup carryover and splitting, <i>Hacer Guner Goren</i></li></ol>
<b>16:00 - 16:30</b>	<b>Coffee Break</b>
<b>16:30 - 17:30</b>	<b>Business Meeting</b> The action plan of the EWG LOT will be discussed, <i>Christian Almeder, Raf Jans and Wilco van den Heuvel</i>
<b>19:00 - 22:00</b>	<b>Porto Cellars + Riversides Dinner</b> The Riverside Dinner is at restaurant Porto Cruz that is located at Largo Miguel Bombarda 23, 4400 - 222 Vila Nova de Gaia.

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## Friday, 29 August

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	<b>Complexity / Approximability</b>
<b>09:30 - 11:00</b>	<ol style="list-style-type: none"><li>1. New results on the lot sizing problem with multi-mode replenishment, <i>Ayse Akbalik and Christophe Rapine</i></li><li>2. A computational study of the local cuts from two-period convex hull closures for big-bucket lot-sizing problems, <i>Ioannis Fragkos and Kerem Akartunali</i></li><li>3. A Theoretical and Computational Study of Two-Period Relaxations for Lot-Sizing Problems with Big Bucket Capacities, <i>Mahdi Doostmohammadi and Kerem Akartunali</i></li></ol>
<b>11:00 - 11:30</b>	<b>Coffee Break</b>
	<b>Sustainability</b>
<b>11:30 - 13:00</b>	<ol style="list-style-type: none"><li>1. Capacitated lot sizing with product substitution in closed-loop supply chains, <i>Kristina Burmeister and Florian Sahling</i></li><li>2. Bi-Level Lot-Sizing Problems with Reusable By-Products, <i>Mehdi Rowshannahad, Nabil Absi, Stéphane Dauzère-Pérés and Bernard Cassini</i></li><li>3. Stochastic capacitated lot sizing in closed-loop supply chains, <i>Timo Hilger, Florian Sahling and Horst Tempelmeier</i></li></ol>
<b>13:00 - 13:30</b>	<b>Closing Session</b>

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## A maritime inventory routing problem with stochastic travelling times

*Agra, Agostinho*<sup>1</sup>  
*University of Aveiro*  
*aagra@ua.pt*

*Christiansen, Marielle*  
*Norwegian University of Science and Technology*  
*mc@iot.ntnu.no*

*Delgado, Alexandrino*  
*University of Cape Verde*  
*alexandrino.delgado@docente.unicv.edu.cv*

*Hvattum, Lars Magnus*  
*Norwegian University of Science and Technology*  
*lars.m.hvattum@iot.ntnu.no*

### Abstract

We consider a stochastic maritime inventory routing problem where a company is responsible for both the distribution of oil products between islands and the inventory management of those products at unloading ports. Ship routing and scheduling is associated with uncertainty in weather conditions and unpredictable waiting times at ports. In this work, both sailing times and port times are considered to be stochastic parameters.

A two-stage stochastic programming model with recourse is presented where the first-stage consists of routing, loading and unloading decisions, and the second stage consists of scheduling decisions. The model is solved using a decomposition approach similar to an L-shaped algorithm where optimality cuts are added dynamically, and this solution process is embedded within the sample average approximation method. A computational study based on ten real-world instances is presented.

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<sup>1</sup>Research supported by national funds through the Fundação Nacional para a Ciência e a Tecnologia, FCT, and Fundo Europeu de Desenvolvimento Regional FEDER, under project INROUTE - EXPL/MAT-NAN/1761/2013.

## 1 Introduction

We consider a short sea shipping problem occurring at the archipelago of Cape Verde where fuel oil products are imported and delivered to specific islands and stored in large supply storage tanks. From these islands, fuel oil products are distributed among all inhabited islands using a small heterogeneous fleet of ships. Products are stored in separate consumption storage tanks with limited capacity. Some ports have both supply tanks for some products and consumption tanks for other products. As the capacities of the supply tanks are very large compared to the total consumption over the planning horizon, the inventory aspects for these tanks can be ignored. Not all islands consume all products. Consumption rates are assumed to be constant over the time horizon.

Each ship has a specified capacity, fixed speed, and cost structure. The cargo hold of each ship is separated into several cargo tanks. The products cannot be mixed, so we assume that the ships have dedicated tanks for the particular products. The ships are either sailing, waiting outside a port or operating (loading or unloading).

At ports, we consider set-up times for the coupling and decoupling of pipes and operation times which depend on the amount loaded or unloaded. Minimum and maximum unloading quantities can be derived. The maximum unloading quantity is imposed by the inventory capacity at the consumption port and by the ship cargo tank capacity.

The driving force in the problem is the need for fuel oil products in the consumption storage tanks. If the demand is not satisfied, the backlogged demand will be penalized by a cost.

The traveling times depend upon the weather conditions and are considered stochastic. The uncertain time parameter at port is mainly related to the time from arrival to start of operation. Hence, a specified waiting time before start of service is defined as stochastic, while the operation times are deterministic.

The inter-island distribution plan consists of routes and schedules for the fleet of ships, and describes the number of visits to each port and the quantity of each product to be loaded or unloaded at each port visit. This plan must satisfy the capacities of the ships and consumption inventories while minimizing the sailing and port costs as well as the expected penalty costs of backlogged demand. There is great flexibility in the route pattern of a ship, such that a ship may visit several loading ports as well as unloading ports in succession and the quantities loaded or unloaded are variable as well as the number of visits at each port.

A deterministic variant of this problem was solved to optimality in [1] for short time horizons. Heuristics for the same problem with time horizons up to 6 months were developed in [2]. Here we describe a stochastic programming model with recourse where the routes and the quantities to load and unload must be fixed *a priori*, that is, before actual values of the uncertain parameters are revealed, while the schedule

of the loading and unloading operations can be adjusted according to the observed sailing and port times.

We propose an optimization approach that combines the use of the sample average approximation method with a decomposition procedure resembling an L-shaped method [3, 4].

## 2 Optimization approach

We consider a two-stage stochastic programming model with recourse. The routes and the quantities to load and unload are determined in the first stage, while the schedule of the loading and unloading operations can be adjusted to the scenario in the second stage. Thus, the inventory level variables are also allowed to change according to the realization of the stochastic parameters. For a given set of  $n$  scenarios  $\{c^j, j = 1, \dots, n\}$  and a first stage solution  $X$ , the objective function can be written as  $z_n(X) = C(X) + \frac{1}{n} \sum_{j=1}^n H(X, c^j)$ , where  $C(X)$  represents the sailing and port operation costs for solution  $X$ , and  $H(X, c^j)$  represents the penalty for the backlogged demand under scenario  $c^j$ .

Since the complete model is too large to be solved efficiently, it is decomposed into a master problem and one subproblem for each scenario, following the idea of the L-shaped algorithm. To solve the master problem for a set of scenarios  $\Omega$ , we first solve to optimality a master problem including only one scenario. Since the model has the property that a feasible solution to the first stage can be completed with a feasible solution to the second stage for each scenario, the resulting values for the first stage decision variables are feasible for the complete problem with all scenarios. However, we need to check whether the solution is optimal for the complete model. To do that we check, for each scenario, whether there is backlogged demand when the deliveries are made as early as possible. If such a scenario with backlogged demand is found, we add to the master problem additional variables and constraints enforcing the backlogged demand to be counted in the objective function. Then the revised master problem is solved again, and the process is repeated until all the optimality constraints are satisfied. Hence, as in the L-shaped method, the master problem initially disregards the recourse cost, and an improved estimation of the recourse cost is gradually added to the master problem by solving optimality subproblems and adding the corresponding cuts. We show that optimality cuts can be derived efficiently by solving longest path problems on an auxiliary acyclic network.

In order to solve the problem with many scenarios, we apply the sample average approximation method [6]. First we consider  $M$  separate sets of scenarios. Each set of scenarios,  $i \in \{1, \dots, M\}$  contains a small number of  $m$  scenarios,  $\{c^{i1}, \dots, c^{im}\}$ . The model is solved for each set of scenarios  $i$  using the decomposition approach described above. Let  $X^i$  denote the obtained first stage solution. The  $M$  candidate

solutions  $X^1, \dots, X^M$ , are then compared using a different, and much larger, set of  $n$  scenarios  $\{\hat{c}^1, \dots, \hat{c}^n\}$ . This larger set of  $n$  scenarios can be regarded as a benchmark scenario set representing the true distribution (see [5]). The best solution is given by  $X^* = \operatorname{argmin}\{z_n(X^i) : i \in \{1, \dots, M\}\}$ .

### 3 Computational tests

We report the results from a computational experimentation based on ten real-world instances, considering two different ships, seven ports, four products, and a time horizon of eight days. The instances differ by the initial inventory levels. To solve the master problem we used the solver Xpress Optimizer. The results have shown that: (i) the running times of the decomposition method are much lower than the running times obtained by solving the complete model. Additionally, the number of times the separation problem (to find optimality cuts) is called is very small and few cuts are added; (ii) by increasing  $m$  the cost of the selected solution decreases but the running times increase substantially; and (iii) the gains for using stochastic programming instead of the deterministic model based on expected values are in general very high.

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# Benders Decomposition applied to cooperative lot-sizing

*Andreas Elias*  
*University of Duisburg-Essen*  
*andreas.elias@uni-due.de*

*Alf Kimms*  
*University of Duisburg-Essen*  
*alf.kimms@uni-due.de*

## Abstract

We consider a special lot-sizing problem in the context of purchasing alliances. We focus on a supply chain which consists of several retailers and one supplier. The retailers are free to cooperate in order to benefit from quantity discounts. In case of a cooperation, transshipments are possible, that is, movement of a product from one retailer to another. A mixed integer programming problem is introduced to cope with the decision problem of who orders when, for whom, how many products and how many products are in stock or being transshipped. Our goal is to minimize the total cost of the system. A Benders Decomposition approach is applied to find a solution to this problem.

**Keywords:** Supply Chain Management, Cooperative Lot-sizing, Benders Decomposition

## 1 Introduction

In times of globalization supply chain management has become a topic of constantly growing importance. Supply Chain managers are forced to think of themselves as members of complex supply chain networks instead of isolated decision makers. In this paper we combine the three different aspects cooperative procurement, quantity discounts and lateral transshipments from the literature. We present a cooperative procurement situation under consideration of quantity discounts and the possibility of lateral transshipments among coalition partners.

Cost efficient procurement can make a substantial difference in terms of competition advantages. Therefore, cooperative procurement can be beneficial for the cooperating parties. In practice, famous examples for purchasing alliances such as

the Renault-Nissan Purchasing Organization have proven to be advantageous. In the literature, game theory is used as a tool to evaluate the goodness of cooperations in supply chain management. For a review of game theoretic approaches in supply chain management we refer to Leng and Parlar [1]. Game theoretic approaches to find fair cost allocations among partners of a coalition in a cooperative procurement application and more literature references in the field of game theory can be found in Drechsel and Kimms [2].

One of the most obvious reasons to participate in cooperative procurement is the exploitation of quantity discounts. A lot of research has been done in the field of quantity discounts in supply chains. Benton and Park [3] give an overview of work that has been published in the years between the mid-seventies and the mid-nineties investigating the influences of different kinds of quantity discounts on lot sizes. Lee et al. [4], Munson and Hu [5], Shinn et al. [6], Choudhary and Shankar [8] or Goossens et al. [7] integrate quantity discounts into other existing problems such as supplier selection problems, carrier selection problems, inventory management or procurement problems.

After products have been ordered they need to be shipped to their destination. Lateral transshipments which often occur in inventory management problems in the literature are a possibility to do so. Therefore, inventory management problems with transshipments are the third field of research which is of higher interest for the work of this paper. Paterson et al. [9] give an excellent review of what has been contributed before 2011 in the field of inventory management considering lateral transshipments. More recent work is that of Özdemir et al. [10], Paul and Yenipazarli [11] or Truschkin and Elbert [12] where production capacities, point process driven demand and impacts of transport policies on the modal split are examined. Hochmuth and Köchel [13] present a simulation optimization approach as an alternative to heuristic solution methods. Coelho et al. [14] take a look at inventory-routing problems and take transshipments into account.

Cooperative procurement, quantity discounts and lateral transshipments are established research fields. But to the best of our knowledge both quantity discounts and lateral transshipments have not been integrated simultaneously in a cooperative procurement setting yet. Transshipments, however, have the potential to intensify the exploitation of quantity discounts and might be necessary in purchasing alliances. Thus, we present a mixed integer programming model from an integrated perspective in order to solve the optimization problem before finding cost allocations. Benders Decomposition from Benders [15] is applied to find a solution. The problem setting is the following:

The underlying problem describes a situation of cooperative procurement where one decision maker needs to find a procurement plan for a system consisting of one supplier and multiple retailers. We assume that all retailers have demand for the same

single product. The retailers however are not equally treated by the supplier. That is, the supplier offers different prices to different retailers depending on their negotiating power. Therefore, it is possible that some retailer  $A$  can order the same amount of products for a lower price than some other retailer  $B$ . In that case, retailer  $B$  could benefit from a cooperation with retailer  $A$ . In a cooperation demands of cooperating retailers are bundled so that retailer  $A$  orders products to satisfy his own demand and that of retailer  $B$ . Afterwards, the products demanded by retailer  $B$  are moved to his location. We call such a movement of products a transshipment. Transshipments are possible between all retailers and there are no capacity restrictions for transshipment quantities. They can be executed in any period before the period of demand satisfaction and after the period of ordering. The strategy of demand bundling leads to higher order quantities. We assume that higher order quantities are rewarded by the supplier by granting all-units quantity discounts. The quantity discount schedule is equal for all retailers. Further, all retailers have inventories with no capacity limits. Products in stock are interpreted as tied-up capital so that the cost of capital evaluated with the help of the actual purchasing prices determines the inventory costs. The decision maker determines all product flows of the system that is, direct orders from the supplier to retailers and transshipments between retailers. Thus, order and inventory quantities are directly affected by his decisions which influence the variable purchasing costs considering quantity discounts and the inventory costs. The goal is to minimize the total costs consisting of variable and fix purchasing costs, inventory costs and transshipment costs.

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# New results on the lot sizing problem with multi-mode replenishment

*Ayşe AKBALIK*

*Université de Lorraine, LGIPM, Ile du Saulcy, Metz, F-57012, France  
ayse.akbalik@univ-lorraine.fr*

*Christophe RAPINE*

*Université de Lorraine, LGIPM, Ile du Saulcy, Metz, F-57012, France  
christophe.rapine@univ-lorraine.fr*

## Abstract

We study the single-item uncapacitated lot sizing problem with multi-mode replenishment and batch deliveries (ULS-MMB). Specifically, we consider that each replenishment mode has an FTL cost structure, incurring a fixed ordering cost plus a fixed cost per batch. This problem arises in practice when a company can order the product units from different suppliers in each period. We show that this problem is NP-hard even when restricted to a single period. We then show that ULS-MMB can be transformed into a lot sizing problem with only one replenishment mode per period, that is, ULS-MMB is a special case of lot-sizing with time-varying batch sizes. This simple observation allows us to improve some results known in the literature of multi-mode replenishment. We also propose a very efficient 2-approximation algorithm and establish that the problem admits an FPTAS.

## 1 Introduction

We consider the single-item uncapacitated lot-sizing problem where a firm can place orders to multiple suppliers and the quantities ordered from a supplier are replenished by batch (*i.e.* truck or container). Each supplier incurs a specific procurement cost, and we assume that any quantity can be ordered from any supplier in each period. In the following we call indifferently a replenishment mode *a supplier* or simply *a mode*. This problem can also be seen as a one vendor-one buyer problem with different transportation modes available to ship the units between them. We consider in the following that the buyer has to satisfy his demands known over a time horizon of  $T$  periods, with the possibility to use up to  $M$  different modes in each period. Shortages are not allowed, that is demand  $d_t$  in period  $t$  has to be satisfied from the available

units in stock at the buyer or/and by ordering units in period  $t$  from one or more suppliers. Carrying inventory at the buyer from period  $t$  to  $t + 1$  incurs a holding cost of  $h_t$  per unit. Ordering some units from supplier  $i$  in period  $t$  incurs a fixed setup cost  $f_{it}$  and a fixed cost per batch  $k_{it}$  in addition to a unit procurement cost  $p_{it}$ . We also denote by  $B_{it}$  the size of the batches used by supplier  $i$  to ship its units in period  $t$ . The ordering cost assumed for each supplier is known in the literature as a Full Truck Load (FTL) cost structure. It is also called stepwise cost or multiple setup cost. The total replenishment cost  $r_{it}(x)$  for an amount  $x$  ordered in period  $t$  with mode  $i$  is given by :

$$r(0) = 0 \quad \text{and} \quad r_{it}(x) = f_{it} + p_{it}x + \lceil x/B_{it} \rceil k_{it} \quad \text{for } x > 0$$

We can formulate problem ULS-MMB as follows :

$$(\text{ULS-MMB}) \quad \left\{ \begin{array}{l} \min \quad \sum_{t=1}^T (\sum_{i=1}^M r_{it}(x_{it}) + h_t s_t) \\ \text{s.t.} \quad s_{t-1} + \sum_{i=1}^M x_{it} = d_t + s_t \quad \forall t = 1, \dots, T \\ \quad \quad x_{it} \in \mathbb{R}_+ \quad \quad \quad \forall i = 1, \dots, M, \quad \forall t = 1, \dots, T \\ \quad \quad s_t \in \mathbb{R}_+ \quad \quad \quad \quad \quad \quad \forall t = 1, \dots, T \end{array} \right.$$

In the formulation  $x_{it}$  represents the quantity ordered from supplier  $i$  in period  $t$  and  $s_t$  the stock level at the buyer at the end of period  $t$ . Wlog, we assume no initial inventory, that is  $s_0 = 0$  and null lead time. This formulation is non-linear because of the FTL procurement cost  $r_{it}(\cdot)$ .

In the literature we have only found the three following studies considering lot sizing from multiple suppliers under multiple setup cost structure: Jaruphongsa *et al.* [5], Eksioğlu [4] and Bai and Xu [2]. Jaruphongsa *et al.* [5] consider the special case of ULS-MMB with only two modes and stationary procurement costs at each retailer, that is  $r_{it}(\cdot) = r_i(\cdot)$  for all periods  $t$ . They establish some dominance properties, and propose polynomial time dynamic programming algorithm for the cases with at most one mode having multiple setup cost structure (other mode can have fixed charge cost structure). Eksioğlu [4] considers also ULS-MMB with stationary costs, but with  $M$  modes. The author proposes a polynomial time algorithm in  $O(MT^2)$  with only fixed charge cost structures using some dominance properties proposed by Jaruphongsa *et al.* [5]. The same author proposes a primal-dual heuristic for the case of  $M$  modes with batch deliveries. Bai and Xu [2] consider some special cases of ULS-MMB with incremental and all-unit quantity discount cost structures and multiple setup costs, and give polynomial and exponential time algorithms for those cases. Notice that in none of these studies the complexity status of problem ULS-MMB is established.

## 2 Transformation to the single-mode problem

Naturally one can think that the multi-mode replenishment problem is more general than the classical single-mode problem. Here we observe that ULS-MMB is also a special case of the lot-sizing problem with only one replenishment mode, namely the uncapacitated lot-sizing problem with batch deliveries and time-varying batch sizes (ULS-B). Specifically, an instance  $I$  of ULS-MMB can be transformed in linear time into an instance  $I'$  of ULS-B over  $MT$  periods as follows: For each period  $t$  of  $I$ ,  $Mt$  periods  $\{M(t-1)+1, \dots, Mt\}$  appear in  $I'$ . The demand is null for all these  $M$  periods except for period  $Mt$  where demand  $d_t$  occurs. The holding cost between those periods is null, and the cost to carry units from period  $Mt$  to  $(Mt+1)$  is  $h_t$ . Finally, the procurement cost for a period  $t' = M(t-1) + i$ ,  $i = 1, \dots, M$ , is equal to  $r_{it}()$ . Notice that the supplier-dependent procurement costs ( $f_{it}, k_{it}, p_{it}$ ) now appear independently over the periods of  $I'$ . This new problem is a “classical” lot sizing problem with only one supplier (one mode) over a time horizon of  $MT$  periods.

There are several natural properties that one could use from this observation. For the special case studied in [5] where the procurement cost does not contain a fixed cost per batch ( $k_{it} = 0$ ), the classical results for the uncapacitated lot sizing problem with setup costs can be used. In particular the zero-inventory-ordering property is dominant and only one mode is used in each ordering period. Wagelmans *et al.* [6] is one of the three studies proposing an algorithm in  $O(T \log T)$  time for setup costs and linear variable costs. As a consequence ULS-MMB can be solved in time  $O(MT \log(MT))$  when fixed cost per batch are null. As stated before, Eksioglu [4] gives a polynomial time algorithm in  $O(MT^2)$  for this case.

## 3 Complexity result

We establish that ULS-MMB is NP-hard even restricted to a single period, with null setup, null unit procurement and null holding costs. That is, the replenishment cost of supplier  $i$  is simply  $r_i(x) = \lceil x/B_i \rceil k_i$ . Notice that this complexity result implies in particular that ULS-MMB is NP-hard for stationary replenishment costs, like the problem considered in [4]. The result uses the equivalence between problem ULS-MMB restricted to a single period and problem ULS-B with only one positive demand, occurring at the last period. This latter problem has been proved NP-hard in Akbalik and Rapine [1].

## 4 Approximation algorithms

It can be shown that the primal-dual heuristic proposed by Eksioglu [4] for problem ULS-MMB with stationary procurement costs at each retailer can perform arbitrar-

ily bad compared to the optimal solution. Thus, we investigate if approximation algorithms exist for problem ULS-MMB, and more generally for problem ULS-B. To propose an efficient approximation, we replace each FTL procurement cost  $r_{it}(x)$  by an affine procurement cost  $a_{it}(x) = f_{it}/2 + (k_{it}/2 + p_{it}/2)x$  for  $x > 0$ . The resulting problem is polynomial, and can be solved in time  $O(MT \log(MT))$ , see Section 2. Since we have the property that  $a_{it}(x) \leq r_{it}(x) \leq 2a_{it}(x)$  for any quantity  $x$ , we obtain a 2-approximation.

Chubanov *et al.* [3] give an FPTAS for a very general class of lot-sizing problems, assuming simply monotone cost structures for procurement and holding costs. One additional assumption is that each cost function is computable in polynomial time for any quantity. Observe that for ULS-MMB, computing  $r_t^*(X)$ , the minimal cost to order a quantity  $X$  in period  $t$  from the set of suppliers, is an NP-hard problem. However, after the transformation, we obtain an instance of problem ULS-B where each procurement cost can clearly be evaluated in constant time. As a consequence, problem ULS-MMB admits an FPTAS.

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# A joint chance-constraint programming approach for a stochastic lot-sizing problem

*Céline Gicquel*

*Université Paris Sud - Laboratoire de Recherche en Informatique  
celine.gicquel@lri.fr*

*Jianqiang Cheng*

*Université Paris Sud - Laboratoire de Recherche en Informatique  
jianqiang.cheng@lri.fr*

*Abdel Lisser*

*Université Paris Sud - Laboratoire de Recherche en Informatique  
abdel.lisser@lri.fr*

## Abstract

We consider a stochastic lot-sizing problem and study a chance-constraint programming formulation involving joint probabilistic constraints. We propose a solution approach based on a "partial sample approximation". This method does not require the use of any additional binary variables and thus leads to significantly reduced computation time as compared to the previously published "sample approximation" method.

## 1 Capacitated lot-sizing with stochastic demand

We consider a single-item capacitated lot-sizing problem over a planning horizon involving  $T$  periods. We denote  $s$  the setup cost,  $h$  the inventory holding cost per unit per period and  $c$  the production capacity. We introduce two sets of decision variables:  $x_t$  represents the quantity produced in period  $t$ ,  $y_t$  is a binary variable which equals 1 in case a setup takes place in period  $t$  and 0 otherwise.

We consider that the demand to be satisfied in period  $t$  is subject to uncertainty and model it as a random variable  $D_t$  following a known probability distribution. The cumulated demand over periods  $1..t$  is denoted  $DC_{1t} = \sum_{\tau=1}^t D_\tau$ . The fact that  $D_t$  is stochastic implies that the inventory at the end of period  $t$ ,  $I_t = \sum_{\tau=1}^t x_\tau - DC_{1,t}$ , is also a random variable. Hence it might be very costly to ensure that it will remain positive for every possible realization of the demand. One possibility is to assume

that the unsatisfied demand is backlogged and to penalize it through backlog costs in the objective function. Another possibility is the use of chance constraints to limit the probability of a stock-out. This is the topic of the present paper.

We thus propose to formulate the problem as follows (see (1)-(4) below). The objective function (1) seeks to minimize the total setup costs and the expected inventory holding costs. Note that we use the same approximation as in [1] to compute the expected inventory holding costs, i.e. we assume  $\mathbb{E}[\sum_{t=1}^T \max(0, I_t)] \approx \mathbb{E}[\sum_{t=1}^T I_t]$ . Constraints (2) ensure that if a production takes place, the corresponding setup cost is incurred and the capacity limit is respected. Constraint (3) is a joint probabilistic constraint which imposes a lower bound (denoted  $p$ ) on the probability that the inventory level remains positive at the end of all the periods of the planning horizon. Constraint (3) can be seen as a service-level constraint ensuring that the risk of a stock-out during the planning horizon is below an acceptable level defined as  $1 - p$ .

$$\left\{ \begin{array}{l} Z^* = \min S \sum_{t=1}^T y_t + h \sum_{t=1}^T \left[ \sum_{\tau=1}^t x_\tau - \mathbb{E}[DC_{1,t}] \right] \quad (1) \\ x_t \leq cy_t \quad \forall t \quad (2) \\ \Pr \left( \sum_{\tau=1}^t x_\tau \geq DC_{1,t}, \forall t \right) \geq p \quad (3) \\ y_t \in \{0, 1\}, x_t \geq 0 \quad \forall t \quad (4) \end{array} \right.$$

To the best of our knowledge, chance-constraint programming formulations involving joint probabilistic constraints have been investigated for lot-sizing problems by only two papers. In [1], a solution approach based on a partial enumeration of the  $p$ -efficient points of the discrete joint distribution of  $(DC_{11}, \dots, DC_{1T})$  is proposed. [2] uses the sample approximation method introduced by [3]: it relies on an approximation of the joint probabilistic constraint by a Monte Carlo sampling of the random vector  $(DC_{11}, \dots, DC_{1T})$  and requires that the obtained production plan do not lead to any stock-out for at least  $p\%$  of the sampled vectors. This method leads to the formulation of a MILP involving one binary variable for each sampled vector. It could thus be solved by a commercial solver such as CPLEX. However the large sample size needed to get a good approximation of the chance constraint leads to a high number of binary variables in the formulation, which poses some computational difficulty in practice. The present paper proposes a way of avoiding this difficulty thanks to an approximation which does not require the introduction of additional binary variables.

## 2 A partial sample approximation approach

The proposed method relies on the assumption that the random variable representing the demand in period 1,  $D_1$ , is independent of the other random variables  $D_2, D_3, \dots, D_T$ . It uses a Monte Carlo sampling similar to the one used by [3]. However we do not apply it on all random variables but only on  $D_2, D_3, \dots, D_T$ : we thus refer to it as a partial sample approximation. More precisely, we introduce the random vector  $\Delta C = (0, DC_{22}, \dots, DC_{2t}, \dots, DC_{2T})$  providing the cumulated demand over periods 2 to  $t$  for each  $t = 1 \dots T$  and randomly generate  $N$  independent and identically distributed sampled vectors  $\Delta C^1, \dots, \Delta C^i, \dots, \Delta C^N$ . This allows us to use the following approximation of the chance constraint (3):

$$\Pr \left( \sum_{\tau=1}^t x_{\tau} \geq DC_{1,t}, \forall t \right) = \Pr \left( \sum_{\tau=1}^t x_{\tau} - \Delta C_t \geq D_1, \forall t \right) \quad (5)$$

$$= \int_{\Delta C} \int_{D_1} \mathbb{1} \left( \sum_{\tau=1}^t x_{\tau} - \Delta C_t \geq D_1, \forall t \right) f_{D_1}(D_1) dD_1 f_{\Delta C}(\Delta C) d\Delta C \quad (6)$$

$$= \mathbb{E} \left[ \int_{D_1} \mathbb{1} \left( \sum_{\tau=1}^t x_{\tau} - \Delta C_t \geq D_1, \forall t \right) f_{D_1}(D_1) dD_1 \right] \quad (7)$$

$$\approx \frac{1}{N} \sum_{i=1}^N \left[ \Pr \left( \sum_{\tau=1}^t x_{\tau} - \Delta C_t^i \geq D_1, \forall t \right) \right] \quad (8)$$

$$\approx \frac{1}{N} \sum_{i=1}^N \left[ \Pr \left( \min_{t=1..T} \left( \sum_{\tau=1}^t x_{\tau} - \Delta C_t^i \right) \geq D_1 \right) \right] \quad (9)$$

Equality (6) relies on the fact that  $D_1$  is independent of  $\Delta C$  so that the density function  $f(DC_{11}, \dots, DC_{1T})$  is equal to the product of the density functions of  $D_1$  and  $\Delta C$ :  $f_{D_1}(D_1) f_{\Delta C}(\Delta C)$ . Equality (7) translates (6) in terms of the expected value of a random variable  $\pi(\Delta C)$  where function  $\pi : \mathbb{R}^T \rightarrow [0, 1]$  is defined as:  $\pi(V) = \int_{D_1} \mathbb{1} \left( \sum_{\tau=1}^t x_{\tau} - V \geq D_1, \forall t \right) f_{D_1}(D_1) dD_1$ . Function  $\pi$  provides, for a given vector  $V$  describing the cumulated demand over periods 2 to  $t$ , the probability that no stock-out occurs during the planning horizon. (8) can be understood as a sample average approximation of the expected value of  $\pi(\Delta C)$ . We denote  $\pi_i = \Pr \left( \sum_{\tau=1}^t x_{\tau} - \Delta C_t^i \geq D_1, \forall t \right)$  the probability that no stock-out occurs during the planning horizon in case the realization of vector  $\Delta C$  is equal to the sampled vector  $\Delta C_t^i$ . The joint probability defining  $\pi_i$  now involves a single random variable  $D_1$  and can be reformulated as in equation (9), which is easier to handle.

In case  $D_1$  is assumed to follow a uniform distribution on interval  $[L_1, U_1]$ , problem

(1)-(4) can be approximated by the following mixed-integer linear program:

$$\left\{ \begin{array}{l} Z^{PSA} = \min S \sum_{t=1}^T y_t + h \sum_{t=1}^T \left[ \sum_{\tau=1}^t x_\tau - \mathbb{E}[DC_{1,t}] \right] \quad (10) \\ x_t \leq cy_t \quad \forall t \quad (11) \\ \frac{1}{N} \sum_{i=1}^N \pi_i \geq p \quad (12) \\ \pi_i \leq \frac{\sum_{\tau=1}^t x_\tau - \Delta C_t^i - L_1}{U_1 - L_1} \quad \forall i, \forall t \quad (13) \\ \pi_i \leq 1 \quad \forall i \quad (14) \\ y_t \in \{0, 1\}, x_t \geq 0 \quad \forall t \quad (15) \end{array} \right.$$

### 3 Preliminary computational results

We provide some preliminary computational results to compare the proposed partial sample approximation to the sample approximation used in [2]. Both approaches lead to the formulation of a MILP, which are solved with the solver CPLEX 12.6. We consider a small lot-sizing problem involving  $T = 10$  periods. We set  $h = 1$ ,  $S = 50$ ,  $c = 100$  and  $p = 0.9$ . The demand  $D_t$  is assumed to follow a uniform distribution  $U(10, 50)$ . The number of samples used in the approximation,  $N$ , is varied from 100 to 10000. For each value of  $N$ , 10 instances are randomly generated.

Table 1 displays the results obtained with both methods. Each line corresponds to the average value over the 10 related instances. *Costs* is the average value of the optimal solution cost, *Prob* is a post-optimization estimation of the probability

$N$	Sample Approx.			Partial Sample Approx.		
	Cost	Prob	Time	Cost	Prob	Time
100	813	0.813	0.4s	878	0.882	0.4s
1000	878	0.887	172s	896	0.904	4.6s
10000	-	-	-	893	0.902	120.4s

- indicates that no guaranteed optimal solution was found within 1 hour of computation.

Table 1: Preliminary computational results



$\Pr\left(\sum_{\tau=1}^t x_{\tau}^* \geq DC_{1,t}\right)$  involved in constraint (3) and *Time* is the computation time needed to solve to optimality the MILP. These results show that the proposed partial sample approximation method leads to significantly reduced computation time as compared the sample approximation method. Additional computational experiments are currently on-going to confirm these preliminary results on larger instances and to extend the approach to the case where the demand is normally distributed.

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# Capacity estimation based on clearing functions for the capacitated lot-sizing problem

*Frank Herrmann*

*Technical University of Applied Sciences Regensburg, Innovation Center for Factory Planning and Production Logistics, Universitätsstr. 31, 93053 Regensburg, Germany  
Frank.Herrmann@OTH-Regensburg.de*

*Frederick Lange*

*Technical University of Applied Sciences Regensburg, Innovation Center for Factory Planning and Production Logistics, Universitätsstr. 31, 93053 Regensburg, Germany  
frederick.lange@ymail.com*

*Christian Almeder*

*European University Viadrina Frankfurt (Oder), Chair of Supply Chain Management, Große Scharrnstr. 59, 15230 Frankfurt (Oder), Germany  
Almeder@europa-uni.de*

## Abstract

Single-stage capacitated lot-sizing problems (CLSP) are essential for many planning tasks in production systems. Due to the hierarchical planning approach, which is widely used in industrial practice and implemented in commercial enterprise resource planning as well as production planning and control systems, a subsequent scheduling is necessary to gain executable production plans. Capacity consumption depends on many factors as, for example, systems workload, machine availability, or yield. Thus, a feasible and optimal solution of the CLSP may lead to unplanned delays after sequencing and processing the orders on the shop floor. Using a so called clearing function (CF), which represents the relationship between the workload and the output of a production system, this capacity consumption in a CLSP is improved in various ways and tested on a real-world application.

## 1 Introduction

In the hierarchical planning approach (capacitated) lot-sizing and scheduling are successive stages using production systems data as well as time periods on different aggregation levels. An interdigitation is reasonable and necessary in order to improve

the results in a hierarchical planning environment (cf. [1]). For the standard CLSP formulation it is assumed that each product is processed on a single production segment so that each product consumes capacity on one (aggregated) resource (cf. [2]). Our test problem, based on an electrical parts manufacturer in Germany, considers a detailed work plan for each product which determines the individual processing steps within a production segment on individual resources on the shop floor level.

The overall capacity consumption of a production order is estimated by its lead time (i.e. the time between the release into the production system and the time of the completion of the order). Measurements show that lead times are varying and highly dependent on the systems workload. The majority of production planning models fail to consider these dependencies (cf. [3]). Due to the lack of detailed information on the waiting times as a result of competing resources and the dependencies of the lead times and the workload, delays in the completion of lots may occur. In recent literature these dependencies are often described by a clearing function (CF) which represents the relationship between the workload and the output of a production system. Hence, we analyze the integration of the concept of clearing functions and the capacity estimation for the lot-sizing as follows:

1. A simulation-based estimation of a product-specific CF is developed.
2. Several integrations of CF into the CLSP and its evaluation on a real-world application as a test problem are analyzed.

## 2 Planning Models

We resort to the classical CLSP formulation (as given in [2]) and describe in the following the relevant extensions. In order to deal with a rolling planning environment we allow backorders which are penalized in the objective function.

All model modifications discussed in the following affect the capacity constraints. Due to the aggregation of machines to a product segment, the available capacity (limited by the capacity constraints) can be greater than the length of the period. Thus, the processing time (including the setup time) of a lot may not exceed the length of a period which limits the feasible lot size. These restrictions are as follows: Parameters:

- $C$ : Available capacity of the resource in a period.
- $d_{k,t}$ : Demand for product  $k$  in period  $t$ .
- $h_k$ : Unit inventory holding cost for product  $k$ .
- $K$ : Number of products.
- $s_k$ : Setup costs for product  $k$ .
- $T$ : Number of Periods.
- $tb_k$ : Capacity needed for producing one unit of product  $k$ .

- $tr_k$ : Capacity needed for setup a lot of product  $k$ .  
 $\epsilon_t$ : Additional capacity consumption in period  $t$  to finish lots, released in prior planning runs.

Decision variables:

- $\gamma_{k,t}$ : Binary variable which indicates whether a setup for product  $k$  occurs in period  $t$ .  
 $q_{k,t}$ : Planned lot of product  $k$  in period  $t$ .  
 $\epsilon_t$ : Additional capacity consumption in period  $t$  to finish lots, released in prior planning runs.

Capacity constraints

$$\sum_{k=1}^K (tb_k \cdot q_{k,t} + tr_k \cdot \gamma_{k,t}) + \epsilon_t \leq C \forall 1 \leq t \leq T \wedge 1 \leq k \leq K. \quad (1)$$

Maximum lot size

$$q_{k,t} \cdot tb_k + tr_k \leq \Delta^{Per} \Rightarrow q_k^{max} = \frac{\Delta^{Per} - tr_k}{tb_k} \forall 1 \leq t \leq T \wedge 1 \leq k \leq K. \quad (2)$$

In the first modification (CLSP\*) we measure the average lead times which occur by using the CLSP for lot-sizing the orders (in a simulation run). We use demand scenarios, which effects typical order situations in the real-world application. These average lead times ( $\mu_{k,s}$ ) replace essentially the process times ( $tb_k$ ) in (1) and (2).

The other two modifications use the CF. The first one CLSP<sup>CF-I</sup> addresses the effect that waiting times reduces the amount of produced pieces within a period. We use CF to estimate the occurring waiting times within the production system depending on the systems workload: For a planned lot  $q_{k,t}$  of a product  $k$  in period  $t$   $CF_k(q_{k,t})$  is the feasible output and  $w_{k,t} = q_{k,t} - CF_k(q_{k,t})$  the amount of pieces ( $w_{k,t}$ ) which could not be processed within a period. Thus,  $w_{k,t} \cdot tb_k$  is the additional capacity consumption for product  $k$ . In total the capacity constraint is modified to:

$$\sum_{k=1}^K (tb_k \cdot q_{k,t} + tr_k \cdot \gamma_{k,t}) + \epsilon_t \leq C - \sum_{k=1}^K (w_{k,t} \cdot tb_k) \forall 1 \leq t \leq T \wedge 1 \leq k \leq K. \quad (3)$$

In addition the value of  $q_k^{max}$  is calculated depending on the systems workload as follows.

$$q_{k,t} \leq q_k^{max} \Rightarrow q_k^{max} = \frac{\Delta^{Per} - tr_k - w_{k,t} \cdot tb_k}{tb_k} \forall 1 \leq t \leq T \wedge 1 \leq k \leq K \quad (4)$$

For the last modification (CLSP<sup>CF-E</sup>) we use two characteristic points of a CF. The relation between workload and output of a production system is linear up to a specific limit and nonlinear beyond that point. We determine this workload limit  $\kappa_k$ , for each product  $k$ . Secondly, the product-specific clearing function  $CF_k$  converges to a maximum output  $O_k$ . We limit the available capacity by  $C = \sum_{k=1}^K tb_k \cdot \kappa_k + tr_k$ ,  $\forall 1 \leq k \leq K$  and the maximum lot size by  $q_k^{max} = \min \{q_k; CF_k(q_k) = O_k\}$ .

### 3 Computational results

The lot-sizing is done in an rolling planning environment. We use a 12-period planning horizon with a 6-period overlap and simulate over 1000 periods in total.

For all products and all WIP inventories between 1 to 500 pieces a simulation run about 1200 periods is executed; note: each product has the same WIP inventory, and 500 is clearly above the maximum output of the production system. This delivers an expected output which is used as value of the CF. Due to this procedure, the CFs are independent to the investigated demand scenarios.

We compared our models with the lot-sizing method of Groff (as a very good setting in a commercial enterprise resource planning system). Taking limited capacity into account by CLSP causes that the amount of delayed orders (PDO) by Groff is between 3,5% and 6,2% higher than the ones by CLSP; the concrete numbers depend on the demand scenario. Just a little bit better than the PDOs by CLSP are the ones by CLSP<sup>CF-I</sup> - the highest improvement is 5,9%. Much lower PDOs are achieved by CLSP\*. The improvements are between 10,7% and 38,6%. All approaches are outperformed by CLSP<sup>CF-E</sup>. Their PDOs are between 40% and 52% lower than the ones by CLSP.

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# Integration of Lot-Sizing and Scheduling Decisions

*Gómez Urrutia, Edwin David*

*Public Research Centre Henri Tudor;*

*École des Mines de Saint-Étienne, CMP - Site Georges Charpak*

*edwin.gomez@tudor.lu; gomez-urrutia@emse.fr*

*Aggoune, Riad*

*Public Research Centre Henri Tudor*

*riad.aggoune@tudor.lu*

*Dauzère-Pérès, Stéphane*

*École des Mines de Saint-Étienne, CMP - Site Georges Charpak*

*dauzere-peres@emse.fr*

## 1 Introduction

In production management, two main activities are lot-sizing and scheduling, which are performed at tactical and operational levels respectively. Lot-sizing problems consist in determining production quantities and periods per product in order to satisfy the demands all along a planning horizon of several days, weeks or months. Scheduling problems concern the definition of the sequence of operations on resources, as well as starting and completion times of operations, in order to execute the production plan. In single-resource floor shops and other systems where resources are not shared between jobs, the scheduling decisions are limited to determining the production periods for each product, i.e. deciding setup variables. Then, the consideration of capacity constraints in the lot-sizing mathematical model is enough to guarantee consistency at scheduling level. In more complex systems, detailed capacity or scheduling constraints are necessary.

In this paper, we focus on lot-sizing problems including capacity constraints and lot-sizing problems with detailed scheduling. The first ones are classified in the literature in small-time bucket problems and big-time bucket problems [2].

## 2 Small time-bucket lot-sizing problems

In this group of problems, we can find:

- The Discrete Lot-sizing and Scheduling Problem (DLSP), which uses an “all or nothing” assumption to limit the capacity consumption of each period, and imposes the production of only one product per period.
- The Continuous Setup Lot-sizing Problem (CSLP), which authorizes the production of only one product per period, using any fraction of the available capacity.
- The Proportional Lot-sizing and Scheduling Problem (PLSP), which allows producing at most two products per period using the whole period capacity and only one resource.

The DLSP and the CSLP are mainly studied in single-level single-resource scenarios. To the best of our knowledge, scarce publications concerning the multi-level DLSP exist and there are not any references related to the multi-level CSLP. Concerning multi-resource systems, some few works consider parallel-machine systems. In the other hand, contributions around the PLSP cover, in a better balanced way, single-level and multi-level problems in single-resource and multi-resource manufacturing systems. Once again, multi-resource systems correspond to parallel-machine floor shops.

As examples of small time bucket problems, we can cite the work of Gicquel *et al.* [4] who model a DLSP with sequence-dependent setup times and costs and the work of Stadler [9], where a multi-level single-resource PLSP with setup crossover is modeled.

## 3 Big time-bucket lot-sizing problems

The big-time bucket lot-sizing problems are:

- The Capacitated Lot-Sizing Problem (CLSP), which does not limit the number of setups per period or the capacity consumption.
- The General Lot-sizing and Scheduling Problem (GLSP), which divides each macro-period on several micro-periods, imposing the hypothesis of the DLSP for each micro-period. Then, apart from lot-sizes, a setup pattern per macro-period is decided.

The CLSP is the most studied lot-sizing problem with capacity constraints. Most of the works studying the CLSP consider single-level single-resource scenarios. However, we can find some works dealing with single-level parallel-machine problems and multi-level single-machine problems. One of the few works studying a lot-sizing problem with scheduling constraints is that of Mateus *et al.* [7], who consider a CLSP with sequence-dependent setup times and costs in a parallel-machine system. A CLSP with sequence-dependent setups in a complex multi-resource system, configured as a job-shop, is modeled and solved using MIP-based heuristic by Mohammadi [8].

Contributions related to the GLSP are also mainly focused on single-level single-resource problems. Nevertheless, we can find some recent works studying a multi-level parallel-machine problem (e.g. [10]). The integrated multi-level GLSP with sequence-dependent setup times in a job-shop environment is modeled by Fandel and Stammen-Hegene [3].

## 4 Lot-sizing problems with detailed scheduling

Scheduling constraints typically considered in lot-sizing problems are those of sequence-dependent setup times and costs, information that can also be considered in the objective function. However, these constraints do not guarantee feasibility of production plans at operational level in complex manufacturing systems. They only allow selecting the sequence that best fits a cost or time criterion. To guarantee feasible production plans, starting and completion times (detailed scheduling) must be considered in the formulation.

A pioneer work dealing with the integrated lot-sizing and scheduling problem in job-shop systems is that of Dauzère-Pérès and Lasserre [1], who propose an iterative approach that solves a lot-sizing problem for a sequence of operations and a scheduling problem for fixed lot sizes. A recent approach solving the integrated lot-sizing and scheduling problem in job-shops systems was proposed by Gómez Urrutia *et al.* [5], who decompose the problem in a set of lot-sizing problems with fixed sequence solved by a Lagrangian heuristic. The sequence is iteratively improved using a Tabu Search metaheuristic. Other approaches, focused on the chemical industry, were proposed by Li and Ierapetritou [6].

## 5 Conclusions

We have introduced a review on lot-sizing problems including capacity or scheduling decisions. Most of the works deal with single-resource problems. Complex manufacturing systems like job-shops and flow-shops have received few attention. Studies on multi-level problems in this kind of systems are almost non-existent in the literature. Opportunities in this field are oriented toward the development of efficient approaches



linking lot-sizing and detailed scheduling decisions in complex systems through integrated approaches. This would overcome the inconsistency of hierarchical approaches used by ERP and APS systems.

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# The capacitated lot sizing problem with setup carryover and splitting

*Hacer Guner Goren*

*Department of Industrial Engineering Pamukkale University, Denizli, Turkey*

*hgoren@pau.edu.tr*

## Abstract

This paper focuses on the capacitated lot sizing problem with two extensions namely setup carryover and setup splitting. The capacitated lot sizing problem finds the production quantities and periods of the products on a single machine. The problem is extended with the concepts of setup carryover and splitting to use the capacity more efficiently. The setup carryover is defined as the production of a product that is continued over from one period to the following without incurring extra setup costs or times. When the setups are relatively long and can not be finished in a period, the option of setup splitting permits to finish it in the consecutive period. Although there have been number of studies considering setup carryover in lot sizing literature, setup splitting is generally disregarded. However, these two concepts complement each other and should be taken into account together. The aim of this paper is to show the importance of the setup splitting in the capacitated lot sizing models.

## 1 Problem Statement

Lot sizing problems are one of the most important production planning problems. They try to answer two main questions. The first one is what amount and the second one is in which periods we should produce in order to minimize the total costs. According to the type of the demand, lot sizing problems can be classified into two as static and dynamic lot sizing problems. Static lot sizing problems deal with stationary demand whereas dynamic lot sizing problems face with dynamic demand changing through the planning horizon. Considering the dynamic lot sizing problem and its features, the decision maker has to properly divide the planning horizon into periods [1]. These periods can be long enough to produce more than one product or they are relatively short so at most two products can be produced. Due to its importance on the efficiency of the production and inventory systems, lot sizing problems are one of the most challenging production planning problems and have been studied for many years with different modelling features. Among these problems, the capacitated lot sizing problem (CLSP) has received a lot of attention from researchers. The CLSP is

a typical example of a big bucket model which takes capacity constraints into account. However, it does not sequence or schedule the products in a period. The main issue in CLSP is the setup operations. Setups in CLSPs can be expressed as fixed fees and/or as setup times with related costs attached. Setups stated as fixed fees include labour, wastage etc. [2]. The capacity lost due to cleaning, preheating, machine adjustments, calibration, inspection, test runs, change in tooling and starting up a new product is considered as setup time [3]. Setup times can be fixed, different for products (i.e. product dependent) or they can depend on the production sequence (i.e. sequence dependent). Setup operations may have an important effect on the lot sizing and scheduling decisions in some production environments when setup times are assumed [4]. To incorporate more real-world features of production environments, different setup settings have been added to CLSP models. Among them, setup carryover and setup splitting are very important. The setup carryover can be defined as maintaining a setup state of a product from one period to the next one. By doing so, it is possible to save on setup costs and capacity along with the decrease in inventory levels. The setup carryover assumption can be found in both small and large bucket models in literature [5], [6], [7]. The setup splitting can be defined as the opportunity to start a setup operation in one period and continue it to the following one [4]. The setup splitting can also be called as the setup crossover as in [4] since the incomplete setup operation crosses over time periods. Another name for the setup splitting is the period-overlapping setup by [8]. Including the setup splitting assumption into CLSP is important especially when the setup times are long. In some production environments such as process industries, setup operations might be performed in more than two periods. With setup splitting option, it possible to increase the flexibility and find better solutions. In such cases where the setup times are significant, the assumption of the setup splitting is necessary to assure the feasibility [9]. The setup splitting complements the setup carryover, however this issue hasn't received the attention of researchers in literature. To fill this important gap in literature, this study focuses on the CLSP with the extensions of setup carryover and splitting. The objective is to show the importance of the setup splitting among periods when long setup times are considered. Therefore, it is possible to use the capacity more efficiently. We study three different models and show the importance of the setup splitting through these models therefore we do not attempt to develop new and faster algorithms to solve this problem.

## 2 Results

In this talk, we show the importance of setup splitting in capacitated lot sizing problems. For this purpose, we study three different models; the first one is the capacitated lot sizing problem; the second one is the capacitated lot sizing problem with setup

carryover and the third one is the capacitated lot sizing problem with setup carryover and setup splitting. Setup splitting plays an important role in developing feasible schedules and increasing flexibility especially when the setup times are long compared to the length of a period. Although the setup carryover and setup splitting complement each other, setup splitting has been mostly ignored in literature due to the complexity of the model. This study addresses this complex problem and aims to fill this important gap in literature. The future research directions in this area can be to propose effective solution approaches to deal with this problem and include different modeling options such as paralel machines, sequence dependent setup times etc. to get more realistic production settings.

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# Extended formulations for static-dynamic uncertainty strategy under various service level measures

*Huseyin Tunc*  
*Hacettepe University, Institute of Population Studies*  
*huseyin.tunc@hacettepe.edu.tr*

*Onur A. Kilic*  
*Hacettepe University, Institute of Population Studies*  
*onuralp@hacettepe.edu.tr*

*S. Armagan Tarim*  
*Hacettepe University, Institute of Population Studies*  
*armagan.tarim@hacettepe.edu.tr*

## 1 Introduction

Stochastic dynamic lot sizing problem aims to determine when and how much to order in time-varying stochastic period demands. As such, it can be referred as the stochastic extension of the well-known deterministic dynamic lot sizing problem [13]. More specifically, this problem considers a single product periodic review finite horizon inventory system. Period demands are independently distributed random variables. The demand distribution may vary from period to period. A fixed ordering cost is incurred once an order is placed. A linear holding cost is incurred for each unit carried forward in inventory. Also, demands not satisfied from inventory are backordered.

In one of the early studies, Bookbinder and Tan investigate this problem and propose the so-called static-dynamic uncertainty strategy [2]. In this strategy, order schedule is fixed once and for all at the beginning of the planning horizon, and order quantities are dynamically determined based upon observations of the actual inventory at each replenishment epoch. This strategy is particularly appealing since it completely eliminates the setup oriented system nervousness that refers to revisions in replenishment schedules. Hence, static-dynamic uncertainty strategy is of practical interest in a variety of settings including material requirement planning, joint replenishments, and shipment consolidation environments for the sake of its rigid replenishment schedule [3, 8, 4, 5]. Due to all of the above mentioned points, it has drawn a great deal of attention in the literature.

A significant progress has been made on the application of static-dynamic uncertainty strategy in stochastic lot sizing in the last decade. The problem has been investigated under penalty cost scheme and various service level measures, namely  $\alpha$ ,  $\beta$ , and  $\beta_c$ .  $\alpha$  service level refers to a lower bound on the non-stockout probability level, whereas  $\beta$ , and  $\beta_c$  service levels refer to lower bounds on the expected proportion of demand satisfied from stock for each replenishment cycle and over the whole planning horizon, respectively.

Simple heuristics for this problem under penalty cost scheme are developed in [1] and [7]. In [9], a deterministic equivalent mixed integer programming model is proposed for the problem under  $\alpha$  service level constraint. This integer programming formulation is adopted by [10], [11], and [6] to take into account penalty cost scheme,  $\beta_c$ , and  $\beta$  service level measures, respectively. Recently, an extended mixed integer programming formulation for the problem under  $\alpha$  service level constraint is developed in [12]. They prove that the extended formulation has a stronger linear relaxation bound than that of [9] and also numerically show that the extended formulation can solve large scale problem instances in very reasonable time.

In this paper, we adopt this extended formulation and present new extended formulations for the static-dynamic uncertainty strategy in stochastic lot sizing problem under  $\beta$  and  $\beta_c$  service level measures, and penalty cost scheme. We conduct an extensive numerical study and illustrate that the extended formulations can solve realistic-sized problem instances in a reasonable amount of time. Our results reveal that the linear relaxation of extended formulations provide very tight bounds which lead to the optimal solutions without requiring excessive number of branching.

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# A computational study of the local cuts from two-period convex hull closures for big-bucket lot-sizing problems

*Ioannis Fragkos*

*Dept. of Management Science, University College London, United Kingdom  
i.fragkos@ucl.ac.uk*

*Kerem Akartunali*

*Dept. of Management Science, University of Strathclyde, United Kingdom  
kerem.akartunali@strath.ac.uk*

## Abstract

We study the big-bucket capacitated lot sizing problem with setup times. We use the novel methodology of Akartunali et al. (2014) that exploits two-period relaxations of the formulation in order to generate inequalities that cut-off the optimal solution of the linear programming relaxation. Our approach applies column generation in an unconventional way, with the master problem being a distance minimizing formulation and the subproblems being combinatorial two-period relaxations of the original problem. We identify a lower bound of the dimensionality of the generated cuts and provide extensive computational experiments that show how the generated bounds compare with other state-of-the-art approaches. Our results show that, for certain classes of problems, the bound improvement is considerable.

## 1 Introduction

The capacitated lot-sizing problem with setup times (CLSP) is a well-studied combinatorial mixed-integer problem of high practical and theoretical importance [7, 6]. The problem is defined over a given discrete and finite time horizon. The goal is to schedule the production amounts of a set of items that has known demands in each time period so that all item demands are covered. Items are produced in a single machine, and a production is possible only after a setup takes place. Each discrete period has finite time capacity, which is consumed by the setup and production processes that take place. Setup operations and production quantities help in inventory carry given, item-specific, fixed and unit costs respectively. The mathematical programming formulation seeks to find a feasible production schedule that minimizes the joint cost of inventory holding and setups. We first define our notation.

*Indices and Sets:*

$NT$  Number of periods

$NI$  Number of items

*Variables:*

$x_t^i$  Production quantity of item  $i$  in period  $t$

$y_t^i$  Setup of item  $i$  in period  $t$  (1 if production occurs, 0 otherwise)

$s_t^i$  Inventory held of item  $i$  at the end of period  $t$

*Parameters:*

$f_t^i$  Fixed cost per setup of item  $i$  in period  $t$

$h_t^i$  Holding cost per unit of item  $i$  from period  $t$  to period  $t + 1$

$d_t^i$  Demand for item  $i$  in period  $t$

$d_{t,t'}^i$  Total demand from period  $t$  to  $t'$ , i.e.,  $d_{t,t'}^i = \sum_{\tau=t}^{t'} d_\tau^i$

$a^i, ST^i$  Per-unit processing/setup time for item  $i$

Then, the CLSP formulation can be written as follows:

$$\min \sum_{t=1}^{NT} \sum_{i=1}^{NI} f_t^i y_t^i + \sum_{t=1}^{NT} \sum_{i=1}^{NI} h_t^i s_t^i \quad (1)$$

$$\text{s.t. } x_t^i + s_{t-1}^i - s_t^i = d_t^i \quad t \in [1, \dots, NT], i \in [1, \dots, NI] \quad (2)$$

$$\sum_{i=1}^{NI} (a^i x_t^i + ST^i y_t^i) \leq C_t \quad t \in [1, \dots, NT] \quad (3)$$

$$x_t^i \leq M_t^i y_t^i \quad t \in [1, NT], i \in [1, \dots, NI] \quad (4)$$

$$y \in \{0, 1\}^{NT \times NI}, x \geq 0, s \geq 0 \quad (5)$$

## 2 Two-period closures and valid inequalities

We introduced in [1] a polytope that can be used to describe subspace relaxation of the CLSP. The structure of the two-period polytope defined over periods  $(t, t + 1)$  and parametrized by period  $k \geq t + 1$ , hereby referred to as  $X_{t,k}^{2PL}$ , is as follows:

$$x_{t'}^i \leq M_{t'}^i y_{t'}^i \quad i = [1, \dots, NI], t' = t, t + 1 \quad (6)$$

$$x_{t'}^i \leq d_{t',k}^i y_{t'}^i + s_k^i \quad i = [1, \dots, NI], t' = t, t + 1 \quad (7)$$

$$x_t^i + x_{t+1}^i \leq d_{t,k}^i y_t^i + d_{t+1,k}^i y_{t+1}^i + s_k^i \quad i = [1, \dots, NI] \quad (8)$$

$$x_{t,k}^i + x_{t+1,k}^i \leq d_{t,k}^i + s_k^i \quad i = [1, \dots, NI] \quad (9)$$

$$\sum_{i=1}^{NI} (a^i x_{t'}^i + ST^i y_{t'}^i) \leq C_{t'} \quad t' = t, t + 1 \quad (10)$$

$$x, s \geq 0, y \in \{0, 1\}^{2 \times NI} \quad (11)$$

We also make a note of the single period relaxations of [3, 4] due to their relevance. We assume that an optimal solution of a linear programming relaxation of CLSP is at hand. For fixed  $t$  and  $k$ , it is possible to use that solution to construct a point  $\bar{p} := (\bar{x}, \bar{y}, \bar{s}) \in \mathbf{R}_+^{5 \times NI}$ , formed by the production and setup variables of periods  $(t, t + 1)$  and the inventory variables of period  $k$ . Our methodology uses an extreme point representation of  $X_{t,k}^{2PL}$  to generate valid inequalities that cut-off  $\bar{p}$ . First, we formulate the problem of finding the point of  $X_{t,k}^{2PL}$  that has the minimum distance from  $\bar{p}$  as a linear program, using the linearizable  $\mathcal{L}_\infty$  norm. If we denote by  $p$  the index of the extreme points of  $X_{t,k}^{2PL}$ , where we dropped the subscripts  $(t, k)$  for ease of notation, then the minimum distance linear program can be cast as follows (associated dual variables of each constraint are given in brackets):

$$\min z_\infty \quad (12)$$

$$\text{s.t. } \sum_p \lambda_p (x_p)_{t'}^i - z_\infty \leq \bar{x}_{t'}^i \quad \forall i, t' = t, t + 1 \quad (\alpha_{t'}^{-i}) \quad (13)$$

$$\bar{x}_{t'}^i \leq \sum_k \lambda_p (x_p)_{t'}^i + z_\infty \quad \forall i, t' = t, t + 1 \quad (\alpha_{t'}^{+i}) \quad (14)$$

$$\sum_p \lambda_p (y_p)_{t'}^i - z_\infty \leq \bar{y}_{t'}^i \quad \forall i, t' = t, t + 1 \quad (\beta_{t'}^{-i}) \quad (15)$$

$$\bar{y}_{t'}^i \leq \sum_p \lambda_p (y_p)_{t'}^i + z_\infty \quad \forall i, t' = t, t + 1 \quad (\beta_{t'}^{+i}) \quad (16)$$

$$\sum_p \lambda_p (s_p)^i - z_\infty \leq \bar{s}_k^i \quad \forall i \quad (\gamma^i) \quad (17)$$

$$\sum_p \lambda_p \leq 1 \quad (\eta) \quad (18)$$

$$\lambda_p \geq 0, z_\infty \geq 0 \quad (19)$$

Upon termination of column generation, and if  $z_\infty > 0$ , a valid inequality that cuts off  $\bar{p} := (\bar{x}, \bar{y}, \bar{s})$  can be generated in the form:

$$\sum_{i=1}^{NI} \sum_{t'=t}^{t+1} [(\alpha_{t'}^{*+i} + \alpha_{t'}^{*-i}) x_{t'}^i + (\beta_{t'}^{*+i} + \beta_{t'}^{*-i}) y_{t'}^i] + \sum_{i=1}^{NI} \gamma^{*i} s_k^i + \eta^* \leq 0 \quad (20)$$

We refer the interested reader for details (including validity proofs and other technical details) to [1]. It is interesting to note that a valid inequality can be generated before completing the column generation process, but it might not be as strong as the one generated upon column generation termination. Also, we investigate the strength of the generated inequalities by establishing tight lower bounds on their dimension.

### 3 Computational Results

Table 1 shows preliminary computational results that were run on the Trigeiro X dataset [7], and compares with the recent algorithms of [5, 2]. For the sake of brevity, we show results for the 10 X sets for which our algorithm had the larger integrality gaps, but report the average of all 34 X sets. Each X set consists of five instances.

Instance Group	Pimentel	PD	X2PL
X11429	9.6	4.99	4.74
X12429	8	3.92	3.56
X11419	10.37	7.07	3.33
X12419	5.97	4.25	2.13
X11229	3.05	2.07	2.01
X12119	1.73	3.46	1.9
X12229	3.24	1.92	1.65
X11119	1.54	3.13	1.62
X11129	2.53	2.87	1.46
X11219	2.38	2.51	1.41
Average	2.09	1.41	0.97

Table 1: Average integrality gaps for the decomposition approaches of [5, 2] and X2PL for the Trigeiro X dataset.

The talk will give an overview of the developed methodology, and will primarily elaborate on the computational results and implementation challenges that the separation algorithm poses. The interested reader is referred to [1] for extensive details of results and discussions.

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# Simultaneous lotsizing and scheduling with scarce setup resources and multiple setups per period

*Copil, Karina*

*University of Cologne, Dep. of Supply Chain Management and Production,  
Albertus Magnus-Platz, D-50932 Cologne, Germany  
copil@wiso.uni-koeln.de*

*Tempelmeier, Horst*

*University of Cologne, Dep. of Supply Chain Management and Production,  
Albertus Magnus-Platz, D-50932 Cologne, Germany  
tempelmeier@wiso.uni-koeln.de*

## Abstract

We present a model for a dynamic capacitated lotsizing problem with sequence dependent setups, linked lotsizes and multiple machines. Setup resources are scarce since setups are carried out by a common setup resource. This means that all operations have to be synchronized in order to avoid overlapping. Multiple setups for the same product per macro-period are possible.

## 1 Introduction

In a German food company as well as in the automobile industry (see for example [1]), we found a special lotsizing problem. Since setup states can be preserved and setups are sequence dependent, sequences have to be determined simultaneously. A unique assignment of products to machines exists which would usually enable an isolated solution of the simultaneous lotsizing and scheduling problem for each production machine, respectively. This approach is not sufficient for the underlying problem, as setups on different production machines are carried out by specialized setup resources, which can perform only one setup at a time. With regard to the common setup resource, setups on all production machines have to be synchronized.

Only few literature exists considering common setup resources. [1] present a model based on the Proportional Lotsizing and Scheduling Problem for one common setup resource. [2] use the big bucket model formulation for the CLSP-L-SD (Capacitated Lotsizing Problem with Linked Lotsizes and Sequence Dependent Setups) proposed by [3] as basic model and extend it by a common setup resource. [4] present a familiar problem as they extend the GLSP (General Lotsizing and Scheduling Problem) and

CLSP-L-SD by tools which are attached to a machine during the whole production process and have to be synchronized, too.

[2] show that their big bucket model formulation is superior to the small bucket model by [1] (e.g. larger problems can be solved). In addition, the model proposed by [2] presents the underlying problem more precisely. Therefore, we use their formulation for further extensions.

In the CLSP-L-SD, it is assumed that only one setup and one production operation can take place per period as a second setup would be associated with unnecessary costs since two lots of the same product can be combined within one period. However, an additional scarce resource (the common setup resource) exists in our case, which restricts the solution space. Regarding high utilizations and/or long setups in combination with a common setup resource, it can make sense to split production quantities and accept additional setups in order to generate feasible production plans.

We present a new model formulation based on [2] which considers multiple setup and production operations for the same product in one macro-period.

## 2 Problem description

A simultaneous lotsizing and scheduling problem is considered. A product can be produced more than once per macro-period. Setups are carried out by a common setup operator and have to be synchronized for all machines and all positions within a macro-period.

Furthermore, the underlying problem has the following characteristics:

- The planning horizon is divided into  $T$  periods.
- $K$  products are produced.
- $M$  production machines are available.
- Each product is uniquely assigned to a single machine ( $k \in \mathcal{K}_m$ ,  $m = 1, 2, \dots, M$ ).
- Capacities of the production machines are limited by  $b_t^m$  ( $t = 1, 2, \dots, T$ ,  $m = 1, 2, \dots, M$ ).
- A setup operation is performed by a setup resource and a setup resource can perform only one setup at a time.
- Each setup operation is linked with sequence dependent time and costs.
- Setup states are carried over to the next period.
- Production of a lot may run over several consecutive macro-periods and positions within one macro-period.

- Backlogging and overtime are not allowed.
- The objective is to minimize the sum of holding and setup costs.

### 3 Solution approach

Based on the model formulation by [3] for the capacitated lotsizing and scheduling problem with sequence dependent setups and linked lotsizes, [2] introduce new variables documenting start and end times for setup operations as well as additional constraints which coordinate setup and production operations on each machine. A binary variable and corresponding equations ensure the synchronization of setups over all machines.

In order to enable multiple setups and production of the same product in a macro-period, a number of positions  $P$  is introduced. Each macro-period is associated with a given number of positions  $p \in \mathcal{P}_t$ , which defines how many times a setup can take place for a product and period at most. Existing variables linked with setup and production operations and corresponding constraints are adapted to this approach. Further adaptations are necessary as all operations have to be coordinated on a particular production machine and over all positions.

The model is solved with IBM ILOG CPLEX 12.6 and the optimization language OPL. Since the problem becomes even more complex by enabling multiple setups, computation times increase in comparison to the model formulation presented by [2]. Therefore, heuristic solution approaches are necessary.

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# Capacitated lot sizing with product substitution in closed-loop supply chains

*Burmeister, Kristina*  
*Leibniz University Hannover*  
*burmeister@prod.uni-hannover.de*

*Sahling, Florian*  
*Leibniz University Hannover*  
*sahling@prod.uni-hannover.de*

## Abstract

We present a new model formulation for a multi-product capacitated lot sizing problem with quality-dependent remanufacturing and product substitution. The returned products possess different quality levels and can be remanufactured to the next better or to a higher one. External demands for new products as well as for remanufactured products depending on the quality are considered. The demand of remanufactured products with a given quality level can also be satisfied by any remanufactured product with a higher quality level or in addition by a new product. However, this substitution is not allowed in the other direction. Furthermore, a solution approach based on column generation is proposed and first numerical results are presented.

## 1 Introduction

In closed-loop supply chains forward-oriented and backward-oriented product flows are considered, i.e., used products can be returned into the supply chain by customers. This offers the opportunity to remanufacture these returned products to sell them again and gain additional profit. In the literature new products and remanufactured products are often regarded as perfect substitutes to fulfill period's demand. However, this assumption does not always hold as customers may refuse to pay the same price for brand new as well as for remanufactured products. Furthermore, different quality levels of returned products are also considered which depend on the customer's product usage. The better the quality level of a returned product, the less is the remanufacturing effort which has to be done. The different quality levels lead to quality-dependent remanufacturing costs and times. Additionally, we also assume a demand for remanufactured products with different quality levels which can arise from different demand markets.

## 2 Model Assumptions

We present a multi-product capacitated lot sizing problem with quality-dependent re-manufacturing options and product substitution. The processes of (re)manufacturing products take place on multiple machines. Each machine can either remanufacture returned products or manufacture new products. Returned products of a given quality level can be remanufactured to the next better quality level or to a higher one. The processing time of remanufacturing a product to a given quality level depends on the initial quality level of the returned product. Furthermore, returned products can also be disposed of. In Figure 1 the considered product flows are illustrated.

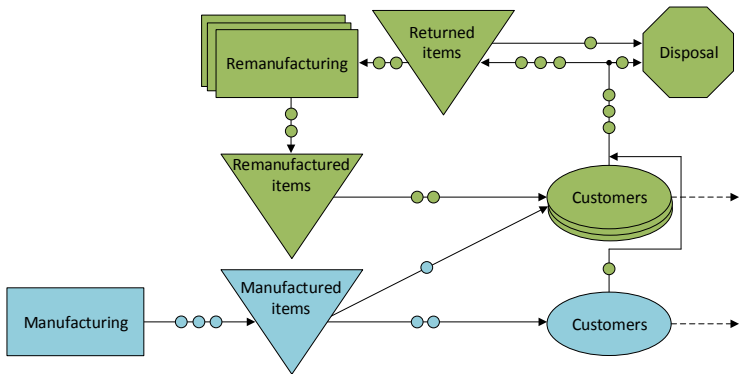


Figure 1: Product flow within the considered closed-loop supply chain.

In each period newly manufactured products as well as remanufactured products with a given quality level are demanded. The external demand of newly manufactured products can only be satisfied by new ones. To fulfill the demand of remanufactured products with a given quality level a downstream substitution is possible, this means a higher quality level can substitute a lower one. In addition, a newly manufactured product can always substitute a remanufactured one.

The objective is to maximize the sales revenue over a given planning horizon with respect to (re)manufacturing, setup, holding, and disposal costs.

As the proposed model can be reduced to the well-known capacitated lot sizing problem (CLSP), this problem is also  $\mathcal{NP}$ -hard. Therefore, a heuristic is necessary to obtain good solutions in reasonable running times.

### 3 Solution Approach

[1] give a structured survey of solution approaches for dynamic capacitated lot sizing problems. They point out, that MP-based heuristics are more flexible in matters of model extensions due to their general procedure. To solve our model the use of an MP-based solution approach seems to be promising as well.

Therefore, we adapt the solution approach based on column generation suggested for the capacitated lot sizing problem with remanufacturing (CLSP-RM) in [2]. First numerical results are presented.

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# A Theoretical and Computational Study of Two-Period Relaxations for Lot-Sizing Problems with Big Bucket Capacities

*Mahdi Doostmohammadi, Kerem Akartunali*

*Dept. of Management Science, University of Strathclyde, Glasgow, UK*

*{mahdi.doostmohammadi,kerem.akartunali}@strath.ac.uk*

## Abstract

In this study, we investigate two-period subproblems proposed by Akartunali et al. (2014). In particular, we study the polyhedral structure of the mixed integer sets related to various two-period relaxations. We derive several families of valid inequalities and investigate their facet-defining conditions. Then we discuss the separation problems associated with these valid inequalities. Finally we investigate the computational strength of these cuts when they are included in a branch-and-cut framework to reduce the integrality gap of the big bucket lot-sizing problems.

## 1 Introduction

Production planning problems have been interesting for both researchers and practitioners for more than 50 years. The problem aims to determine a plan for how much to produce and stock in each time period during a time interval called planning horizon. It is an important challenge for manufacturing companies because it has a strong impact on their performance in terms of customer service quality and operating costs. In this study, we focus on multi-level, multi-item production planning problems with big bucket capacities, i.e., each resource is shared by multiple items and hence different items can be produced in a specific time period. These real-world problems remain challenging to solve to optimality as well as to obtain strong bounds.

Let  $NT$ ,  $NI$  and  $NK$  be the number of periods, items, and machine types, respectively. We assume that each machine type operates only on one level, and each level can employ a number of machine types. The set  $endp$  indicates all end-items, i.e. items with external demand; the other items are assumed to have only internal demand. Let  $x_t^i$ ,  $y_t^i$ , and  $s_t^i$  represent production, setup, and inventory variables for item  $i$  in period  $t$ , respectively. The setup and inventory cost coefficients are indicated by  $f_t^i$  and  $h_t^i$  for each period  $t$  and item  $i$ . The parameter  $\delta(i)$  represents the set of immediate successors of item  $i$ , and the parameter  $r^{ij}$  represents the number of items

required of  $i$  to produce one unit of item  $j$ . The parameter  $d_t^i$  denotes the demand for end-product  $i$  in period  $t$ , and  $d_{t,t'}^i$  is the total demand between  $t$  and  $t'$ . The parameter  $a_k^i$  represents the time necessary to produce one unit of  $i$  on machine  $k$ , and  $ST_k^i$  is the setup time for item  $i$  on machine  $k$ , which has a capacity of  $C_t^k$  in period  $t$ . Let  $M_t^i$  represent the maximum number of item  $i$  that can be produced in period  $t$ . Following the notation of [2], the multi-level, multi-item production planning problems with big bucket capacities can then be formulated:

$$\begin{aligned}
 \min \quad & \sum_{t=1}^{NT} \sum_{i=1}^{NI} f_t^i y_t^i + \sum_{t=1}^{NT} \sum_{i=1}^{NI} h_t^i s_t^i \\
 \text{s.t.} \quad & s_{t-1}^i + x_t^i = s_t^i + d_t^i, & t \in [1, NT], i \in \text{endp}, & (1) \\
 & s_{t-1}^i + x_t^i = s_t^i + \sum_{j \in \delta(i)} r^{ij} x_t^j, & t \in [1, NT], i \in [1, NI] \setminus \text{endp}, & (2) \\
 & \sum_{i=1}^{NI} (a_k^i x_t^i + ST_k^i y_t^i) \leq C_t^k, & t \in [1, NT], k \in [1, NK], & (3) \\
 & x_t^i \leq M_t^i y_t^i, & t \in [1, NT], i \in [1, NI], & (4) \\
 & y \in \{0, 1\}^{NT \times NI}, x \geq 0, s \geq 0. & & (5)
 \end{aligned}$$

Here, (1) and (2) are flow conservation constraints for end-items and intermediate items respectively. The constraints (3) are the big bucket capacity constraints, and (4) guarantee that the setup variable is equal to 1 if production occurs. Finally, (5) give the integrality and non-negativity constraints.

We note that uncapacitated relaxation and single-item relaxation have been studied previously by [5]. In addition, [4] introduced and studied the single-period relaxation with preceding inventory, where they also derived cover and reverse cover inequalities for this relaxation. Finally, we also remark the work of [3] on a single-period relaxation as a relevant study.

## 2 Two-Period Relaxation

Now, we present the feasible region of a two-period, single-machine relaxation of the multi-level, multi-item production planning problems with big bucket capacities, denoted by  $X^{2PL}$  (see [1] for details).

$$\begin{aligned}
 x_{t'}^i &\leq \widetilde{M}_{t'}^i y_{t'}^i, & i \in \{1, \dots, NI\}, t' = 1, 2, \\
 x_{t'}^i &\leq \widetilde{d}_{t'}^i y_{t'}^i + s^i, & i \in \{1, \dots, NI\}, t' = 1, 2, \\
 x_1^i + x_2^i &\leq \widetilde{d}_1^i y_1^i + \widetilde{d}_2^i y_2^i + s^i, & i \in \{1, \dots, NI\},
 \end{aligned}$$

$$\begin{aligned}
 x_1^i + x_2^i &\leq \tilde{d}_1^i + s^i, & i &\in \{1, \dots, NI\}, \\
 \sum_{i=1}^{NI} (a^i x_{t'}^i + ST^i y_{t'}^i) &\leq \tilde{C}_{t'}, & t' &= 1, 2, \\
 x &\geq 0, s \geq 0, y &\in \{0, 1\}^{2 \times NI}.
 \end{aligned}$$

Since we consider a single machine, we dropped the  $k$  index from this formulation, however, all parameters are defined in the same lines as before. Observe that the obvious choice for the horizon would be  $t + 1$ . In this case, the definition of the parameter  $\tilde{M}_{t'}^i$  is the same as of the basic definition of  $M_{t+t'-1}^i$ , for all  $i$  and  $t' = 1, 2$ . Similarly, capacity parameter  $\tilde{C}_{t'}$  is the same as  $C_{t+t'-1}$ , for all  $t' = 1, 2$ . Cumulative demand parameter  $\tilde{d}_{t'}^i$  represents simply  $d_{t+t'-1, t+1}^i$  (or echelon demand in case of subassemblies with no external demand), for all  $i$  and  $t' = 1, 2$ , i.e.,  $\tilde{d}_1^i = d_{1,2}^i$  and  $\tilde{d}_2^i = d_2^i$ .

Next, we remark the following polyhedral result for  $X^{2PL}$  (see [1] for details).

**Proposition 2.1** *Assume that  $\tilde{M}_t^i > 0, \forall t \in \{1, \dots, NT\}, \forall i \in \{1, \dots, NI\}$  and  $ST^i < \tilde{C}_t, \forall t \in \{1, \dots, NT\}, \forall i \in \{1, \dots, NI\}$ . Then  $\text{conv}(X^{2PL})$  is full-dimensional.*

In this study, we investigate the case of  $ST^i = 0, \forall i \in \{1, \dots, NI\}$ . Then two relaxations of  $X^{2PL}$  are established and their polyhedral structures are studied. We present the trivial facet-defining inequalities and then we derive several classes of valid inequalities such as cover and reverse cover inequalities for those mixed integer sets and establish conditions on them to define facets. Next, we discuss on the separation problems associated to those valid inequalities. Lastly, we include these cuts in the branch-and-cut algorithm to improve the integrality gap of the multi-level, multi-item production planning problems with big bucket capacities. This talk will cover the details of these aspects.

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## Competitive uncapacitated lot-sizing game

*Margarida Carvalho*  
*University of Porto*  
*margarida.carvalho@dcc.fc.up.pt*

*Claudio Telha*  
*University of Louvain*  
*claudio.telha@uclouvain.be*

*Mathieu Van Vye*  
*University of Louvain*  
*mathieu.vanvyve@uclouvain.be*

### Abstract

We consider a game where there is a separate market (for a single product) in each of a given set of discrete time periods. Each player can decide the quantity of product to place in each time period (market). Each player has her own production facility which is represented as an uncapacitated lot-sizing model (i.e. production incurs fixed cost and inventories are allowed). We show that this game is potential and always admits at least one pure Nash Equilibrium. There are in general several such equilibria. When there are no variable production and inventory cost, we can compute one pure Nash Equilibrium in polynomial time. When there is only one period, it is NP-Hard to find an optimal pure Nash equilibrium (i.e. with respect to a given objective function), but one can compute such an optimal equilibrium in pseudo-polynomial time.

## 1 Introduction

We consider the situation where several producers compete for a single-product market over time. Our goal is to understand what is the likely behaviour of these respective producers (how they will produce and when, how much they will place in the market and when) and what will the resulting market prices will be. The natural conceptual framework for such a study are Game Theory and the key concept of a Nash Equilibrium.

The model we build has a discretized finite time horizon with  $T > 0$  periods. In each period  $t$  there is a market for the single product. We assume that in a single



period  $t$ , everyone buys and sells at the same market price  $p_t$ . The market in period  $t$  is represented by the demand function  $p_t = a_t - b_t q_t$  where  $p_t$  is the market price,  $q_t$  is the total quantity placed in the market, and  $a_t, b_t$  are given parameters. We assume that  $m > 0$  producers (or players) compete for this multi-period market. The production structure of each producer is represented by an uncapacitated lot-sizing model. That is, each producer  $i$  has to decide how much to produce in each of the time period  $t$  (production variable  $x_t^i$ ) and how much to place in the market (variable  $q_t^i$ ). They are fixed and variable (linear) production costs, no upper limit on production quantities, and a producer can build inventory by producing in advance. We assume without loss of generality that there is no inventory cost. So we obtain the following model.

Each player  $i = 1, \dots, m$  solves the following parametric programming optimization problem.

$$\max_{y^i, x^i, q^i, h^i} \Pi^i(x, y, q) = \sum_{t=1}^T (a_t - b_t \sum_{j=1}^m q_t^j - c_t^i) q_t^i - \sum_{t=1}^T F_t^i y_t^i \quad (1a)$$

$$\text{subject to } x_t^i + h_{t-1}^i = h_t^i + q_t^i \quad \text{for } t = 1, \dots, T \quad (1b)$$

$$0 \leq x_t^i \leq M y_t^i \quad \text{for } t = 1, \dots, T \quad (1c)$$

$$h_0^i = h_T^i = 0 \quad (1d)$$

$$y_t^i \in \{0, 1\}, h_t^i \geq 0 \quad \text{for } t = 1, \dots, T, \quad (1e)$$

where  $h_t^i$  is the inventory of producer  $i$  at the end of period  $t$ ,  $M$  is a large constant,  $c_t^i$  and  $F_t^i$  are the unit (variable) and fixed production cost respectively and  $y_t^i$  is a binary variable indicating whether there is positive quantity produced in period  $t$  by player  $i$ .

We call this an Uncapacitated Lot-Sizing Game or ULSG in short. A pure Nash Equilibrium (pNE in short) is an assignment to the variables  $x, y, q, h$  so that they are indeed optimal solution to the parametric program (1a)–(1e).

## 2 Literature Review

Most of the literature about lot sizing games focuses in the cooperative direction. In this type of games instead of searching for a Nash equilibrium, the goal is to find coalitions between the players such that they do not have incentive to leave it (that would traduce in an utility decrease). See for example [5].

The authors of [1] study a Bertrand and Cournot competition game with one period. Their Cournot competition model is similar to our one period lot sizing game, but their market price functions are more general (for example, the parameters  $a$  and  $b$  can be different for each player).

On the paper [2] a Bertrand price competition game is analyzed. Here, the problem is similar to our lot sizing game (if we invert the demand function). The main difference is in the players' strategies. Each player has as strategy a selection of a market price which is maintained throughout the game. Given the market price of each player the demands in each time period for each player are determined. The authors are able to get sufficient conditions for the existence and efficiency of computing one Nash equilibrium if the fixed costs are constant during the all time horizon for each player.

Hongyan Li and Joern Meissner [4] consider a lot sizing game in which the players' strategies are the production capacities purchased in the beginning of the time horizon and a lot-size plan. The cost of buying capacity depends on the total capacity purchased by the players. After this choice, the players' problem is just a single-item lot sizing problem with limited capacity. The authors prove the existence of a capacity equilibrium under modest assumptions.

The authors of [3] built a methodology to solve Cournot oligopolies when part of the players' decision variables are integer. Typically, in the continuous game, under some assumptions (e.g. players' objective function is convex and their constraints are linear), Karush-Kuhn-Tucker conditions are enough to establish each player optimization problem and thus, compute a Nash equilibrium for the game. However, the presence of discrete variables makes the KKT-conditions not valid. In this way, the authors propose a new approach that provides a compromise between complementarity and integrality, showing that when there is a Nash equilibrium satisfying the integrality requirement it can be found.

### 3 Contribution

We are interested in the following questions related to this game. Does there always exist one pNE for this game? Can there be multiple pNE? Can we compute one (any) pNE in polynomial time (if such a pNE exists)? Can we optimize a function linear in the variables over the set of pNE in polynomial time? We are able to partially answer these questions by proving the following results.

**Proposition 1** *ULSG is a potential game, and the maximizer of the following potential function*

$$\begin{aligned} \Phi(x, y, q) &= \sum_{t=1}^T \sum_{p=1}^m -F_t^p y_t^p - c_t^p x_t^p + \left( a_t - \frac{b_t}{2} (2q_t^p + \sum_{j \neq p} q_t^j) \right) q_t^p \\ &= \sum_{p=1}^m \left( \Pi^p(x, y, q) + \sum_{t=1}^T \frac{q_t^p b_t}{2} \sum_{j \neq p} q_t^j \right) \end{aligned}$$

is always a pNE. This implies that a pNE always exists for the ULSG.

This justifies the fact that we only consider pure Nash Equilibria, and not mixed strategies (where players are allowed to choose among the possible strategies with some given probability, instead of selecting only one).

**Proposition 2** *When  $c = 0$ , ULSG is a congestion game, and maximizing the potential function  $\Phi$  is equivalent to solving a minimum-cost network flow problem that can be solved in polynomial time. Therefore finding a pNE for ULSG when there are no variable costs can be done in polynomial time.*

**Proposition 3** *Even when  $T = 1$  and the objective function is given by  $\sum_{i=1}^m f^i y^i$ , optimizing over the set of pNE is NP-Hard.*

This is proved by showing that optimizing over the set of pNE is at least as hard as Subset-Sum.

**Proposition 4** *When  $T = 1$ , one can optimize over the set of pNE in pseudo-polynomial time.*

**Proposition 5** *When  $F = 0$ , the set of pNE of ULSG is a polytope. Furthermore there is only one pNE unless the problem is degenerate.*

The main question left open is whether one can compute a pNE in polynomial time in the general case. Another question is whether the existence of a pNE extends when the lot-sizing model includes capacities or backlogging.

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## Bi-Level Lot-Sizing Problems with Reusable By-Products

*Rowshannahad, Mehdi*

*Ecole des Mines de St-Etienne, CMP, Department of Manufacturing Sciences and Logistics, CNRS UMR 6158 LIMOS, site Georges Charpak, 13541 Gardanne, France  
rowshannahad@emse.fr*

*Soitec, Parc Technologique des Fontaines, 38190 Bernin, France*

*Absi, Nabil*

*Ecole des Mines de St-Etienne, CMP, Department of Manufacturing Sciences and Logistics, CNRS UMR 6158 LIMOS, site Georges Charpak, 13541 Gardanne, France  
absi@emse.fr*

*Dauzère-Pérès, Stéphane*

*Ecole des Mines de St-Etienne, CMP, Department of Manufacturing Sciences and Logistics, CNRS UMR 6158 LIMOS, site Georges Charpak, 13541 Gardanne, France  
dauzere-peres@emse.fr*

*Cassini, Bernard*

*Soitec, Parc Technologique des Fontaines, 38190 Bernin, France  
bernard.cassini@soitec.com*

### Abstract

In this research, we study bi-level lot-sizing problems in which reusable by-products are generated during production. The generated by-products can be reused as raw materials after further processing. However, by-products can be “recycled” only a given number of times. The industrial problem concerns the supply chain of “SOI” (Silicon-On-Insulator) fabrication units. Based on the industrial model, an uncapacitated single-item version of the problem is defined and analyzed. Numerical experiments of both problems are discussed.

## 1 Introduction

Silicon-on-Insulator (SOI) wafers are used instead of silicon-only wafers in semiconductor manufacturing for higher performance and efficiency. SOI wafers can be produced using different technologies. Among them, *Smart-Cut™ Technology* is a very economic and efficient way of fabricating SOI wafers. Using this technology, a thin crystalline layer of a so-called donor wafer (Top wafer) is laid on another silicon wafer (called the handle or Base Wafer) using bonding and splitting processes [2]. As only a thin layer of the Top wafer is transferred to the final product (SOI wafer), it can be reused to produce other SOI wafers. Before reusing the used Top wafer in the SOI production line, it must be reprocessed.

The purchased raw material -which is not yet used in production- is called “Fresh Wafer”. The generated by-product during production is called “Negative Wafer”. The Negative Wafers are not reusable immediately as raw material. The process of making by-products reusable is called “Refreshing”. The refreshed Negative Wafer which can be used again as raw material in production is called “Refresh Wafer”. A purchased raw material (Fresh Wafer) has a limited refresh life. It means that the generated by-product (Negative Wafer) can be reused (refreshed) a limited number of times. The supply chain is depicted in Figure 1.

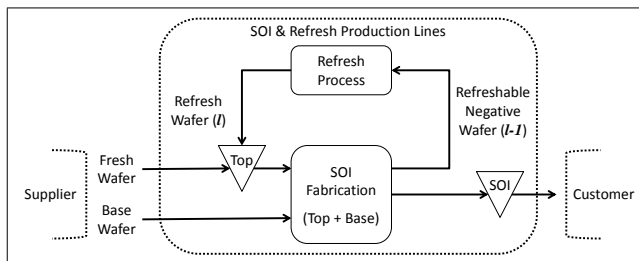


Figure 1: Supply chain of a SOI fabrication unit using the Smart-Cut™ Technology

## 2 Literature Review

Existing concepts in the lot-sizing literature, such as “by-product”, “co-product”, “remanufacturing”, and “recycling” are close to this research. Let us distinguish the new aspects of our research. The problem studied is bi-level, with a parent-component relationship in the item structure between raw materials and the final product. The raw material once used for production is considered as “by-product”. The process of restoring the generated by-products makes them reusable again as raw materials. Therefore, this process can be considered as a “remanufacturing process”. To the best of our knowledge, our problem is not addressed in the literature.

In the literature, the difference between by-product (by-production) and co-product (co-production) is not clear. Both co-products and by-products are generated during production process or at each production run. Co-products have their own independent demand and cannot be used interchangeably. Whereas by-products are undesired “sub-products” generated during production. They must normally be reworked before being able to reuse them.

A multi-item uncapacitated lot-sizing problem in which co-products are produced at each production run is treated in [1], where co-products have their own demand and cannot fulfill the demand of the main product. Other studies consider the co-production of a range of products with different performances in a single production run. The co-products are then sorted according to their key performance to satisfy

demands of each co-product [6]. Remanufacturing in reverse logistics is considered in [4], where there is not only a one-way flow of the products to the customers but materials and products may be returned to the manufacturer for remanufacturing and reuse. This is what is called a “closed-loop supply chain”. Spengler et al. [5] propose mathematical planning models for recycling generated by-products during production, and dismantling and recycling products at the end of their lifetime. Several other studies use the term remanufacturing to denote the restoring or recycling of products [3]. The term remanufacturing is used differently in the literature compared to our industrial problem in which remanufacturing is part of the manufacturing process where the customer is not involved.

### 3 Problem Modeling and Analysis

Based on the industrial problem, a reduced problem is defined with only a single final product and refresh reference. The complete model with procurement, refresh and production setup constraints and related costs will be presented in the workshop. Here, we present the main idea to model the remanufacturing process of the by-products in the refresh process of the reduced single-item model. In order to model the flow conservation constraints, we define two parameters  $T$  and  $d_t$ , which stand respectively for the number of periods and the demand at period  $t$ . The parameter  $l$  indicates how many times the raw material has been used and how many more times it can be used to be fully amortized and reach its maximum refresh level  $l^{max}$ .  $l = 0$  indicates that the raw material is a Fresh Wafer (has never been used), and  $l > 0$  indicates that the raw material is a Refresh Wafer.

The decisions variables are:

- $x_t^l$  Production quantity using Top Wafer at level  $l$  in period  $t$ ,
- $p_t$  Quantity of Fresh Wafer purchased in period  $t$ ,
- $z_t^l$  Quantity of Top Wafer at level  $l$  to be refreshed in period  $t$ ,
- $s_t$  Finished goods (SOI) inventory in period  $t$ ,
- $s_t^{l+}$  Top (Fresh or Refresh) Wafer inventory at level  $l$  in period  $t$ ,
- $s_t^{l-}$  Negative Wafer inventory at level  $l$  in period  $t$ .

The flow conservation constraints of the single-item bi-level lot-sizing problem with reusable by-products are expressed as follows:

$$s_{t-1} + \sum_{l=0}^{l^{max}} x_t^l = d_t + s_t \quad \forall t \quad (1)$$

$$s_{t-1}^{l+} + p_t = x_t^l + s_t^{l+} \quad \forall t, l = 0 \quad (2)$$

$$s_{t-1}^{l+} + z_{t-1}^{l-1} = x_t^l + s_t^{l+} \quad \forall t, l > 0 \quad (3)$$

$$s_{t-1}^{l-} + x_{t-1}^l = z_t^l + s_t^{l-} \quad \forall t, l \quad (4)$$

Constraint (1) models the finished goods (SOI) flow conservation while Constraint (2) models the purchased raw material (Fresh Wafer) flow conservation. Each time a production takes place in period  $t$ , ( $x_t^f > 0$  in Constraint (1)), a Negative Wafer is generated. The generated Negative Wafer enters the Negative flow conservation Constraint (4). If the Negative Wafer has not yet reached its maximum refresh level, it can possibly be refreshed. Constraint (3) models the Refresh Wafer flow conservation.

During the workshop, the detailed industrial model and some numerical experiments will be presented. Properties of the single-item version of the industrial model will be discussed.

## Acknowledgments

This study has been done within the framework of a joint collaboration between SOITEC (Bernin, France), and the CMP of the École des Mines de St-Étienne (Gardanne, France). The authors are grateful to the Supply Chain Department of SOITEC for their full support and would like to thank the ANRT (*Association Nationale de la Recherche et de la Technologie*) which has partially financed this study.

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# Worst case analysis of relax-and-fix heuristics for lot-sizing problems

*Nabil Absi*

*Ecole des Mines de Saint-Etienne, CMP Georges Charpak, Gardanne, France*

*absi@emse.fr*

*Wilco van den Heuvel*

*Econometric Institute, Erasmus University Rotterdam, Rotterdam, The Netherlands*

*wvandenheuvel@ese.eur.nl*

## Abstract

Several researchers have proposed so-called MIP based heuristics, such as relax-and-fix and fix-and-optimize, to solve several variants of lot-sizing problems. Computational results show that these MIP based heuristics perform well on sets of randomly generated problem instances. In this research, we make a first attempt to analyze the worst case behavior and we will present some preliminary results

## 1 Introduction

In recent years several researchers have applied MIP based heuristics to lot-sizing problems. In general, a MIP based heuristics uses a MIP formulation of the lot-sizing problem under consideration as a starting point. Since the problem can often not be solved to optimality in a reasonable amount of time, the integrality of a subset of the variables is relaxed and this relaxed problem is solved. Using the information of the resulting solution, some variables are fixed and the problem with the remaining unfixed variables is reoptimized. This procedure continues until all variables are fixed.

Different types of MIP based heuristics, such as fix-and-optimize and relax-and-fix, are proposed to solve complex lot-sizing problems. Typically, these lot-sizing problems have multiple items, capacities, setup times, setup carry-overs, scheduling decisions, etc. We mention the papers [1], [3], [2] and [4] as examples, but this list is far from complete.



## 2 Problem setting and research question

A commonly used MIP based heuristic is the so-called relax-and-fix heuristic (also called fix-and-relax heuristic). Formally, this heuristic works as follows. Consider a MIP formulation of some (lot-sizing) problem and let  $y_j$  be the integer variables indexed by  $j \in J$ . In iteration  $k$ , we have a set  $I_k \subset J$  corresponding to the integer variables that will be optimized over, and a set  $F_k \subseteq I_k$  corresponding to the integer variables that will be fixed at the end of the iteration. For convenience, let

$$\bar{F}_k = \bigcup_{i=1, \dots, k} F_i \text{ and } \bar{I}_k = \bigcup_{i=1, \dots, k} I_i.$$

Starting with  $k = 1$  and  $F_0 = \emptyset$ , the following steps are performed in a relax-and-fix heuristic:

**Step 1** construct problem  $(P_k)$  where the variables  $y_j$  with  $j \in \bar{F}_{k-1}$  are fixed,  $y_j$  with  $j \in \bar{I}_k - \bar{F}_{k-1}$  are integer, and  $y_j$  with  $j \in J - \bar{I}_k$  are relaxed (to continuous variables)

**Step 2** solve  $(P_k)$  and fix the integer variables  $y_j$  with  $j \in F_k$  to the optimal values found

**Step 3** while not all variables are fixed, set  $k = k + 1$ , and go to Step 1

Clearly, to have a valid algorithm with  $n$  iterations, we must have  $\bar{I}_n = J$  and the sets  $F_k$  ( $k = 1, \dots, n$ ) should partition  $J$ .

Several decisions need to be made when implementing a relax-and-fix heuristic for a lot-sizing problem. For example, one needs to decide on (i) which formulation to use (for example, an aggregate, shortest path, or facility location formulation), (ii) how to decompose the problem (for example, decompose over the items or over the periods), and (iii) how to choose the relaxing strategy (for example, how to set the amount of overlap, i.e., the size of  $|I_{k-1} \cap I_k|$ ). Clearly, all these decisions effect the performance of the relax-and-fix heuristic.

As mentioned in the introduction, quite some research has been done on the performance of relax-and-fix heuristics by testing it on sets of randomly generated instances. However, to the best of our knowledge, the worst case behavior of relax-and-fix heuristics has not been investigated so far. This is the main goal of our research.

## 3 Preliminary results

Our starting point is the simplest case possible: a  $T$  periods single-item lot-sizing problem with time-invariant cost parameters using an aggregate formulation. Furthermore, we set  $I_k = F_k = \{k\}$  for  $k = 1, \dots, T$ , which means that we have a

forward rolling and non-overlapping time window of size 1 over the iterations. Our preliminary results are the following:

1. When using a bad big- $M$  (so one that is large) in the aggregate formulation, the relax-and-fix heuristic reduces to the Freeland-Colley heuristic.
2. Even when using a tight big- $M$ , the worst case ratio of the relax-and-fix heuristic is unbounded.

Furthermore, we found instances with the following counterintuitive properties:

1. When increasing the setup cost, the length of the first order cycle decreases.
2. When increasing the size of the time window, the performance of the relax-and-fix heuristic gets worse.

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## An a priori tour-based three phase heuristic for the production-routing problem

*Oğuz Solyali*

*Middle East Technical University, Northern Cyprus Campus, Kalkanlı, TRNC,  
Mersin 10, Turkey  
solyali@metu.edu.tr*

*Haldun Sural*

*Department of Industrial Engineering, Middle East Technical University, Ankara  
06800, Turkey  
sural@ie.metu.edu.tr*

*Jean-François Cordeau*

*CIRRELT and Canada Research Chair in Logistics and Transportation, HEC  
Montréal, Montréal H3T 2A7, Canada  
jean-francois.cordeau@hec.ca*

*Raf Jans*

*GERAD and HEC Montréal, Montréal H3T 2A7, Canada  
raf.jans@hec.ca*

The integrated optimization of production, inventory and distribution routing decisions in supply chains offers tremendous cost saving opportunities to firms. In order to realize these savings, one has to solve a production-routing problem (PRP), which jointly considers production, inventory and distribution routing decisions.

The PRP can be seen as an integration of two-level lot sizing and vehicle routing problems, which are very difficult problems even separately. Thus, the PRP is a very complex problem. Because of its complexity, there are only a few exact algorithms (see [3], [6], [8]) available in the literature for the PRP. On the other hand, many heuristics have been proposed for the PRP with the most successful ones being proposed by Absi et al. [1], Adulyasak et al. [2], and Archetti et al. [6]. For a recent review on the PRP, interested readers can refer to Adulyasak et al. [4].

We consider a production-distribution system in which a supplier orders (or produces) a single product and distributes to  $N$  retailers over a finite time horizon with a fleet of vehicles. Each retailer faces external customer demand in each discrete time period and may keep inventory to meet the demand without backordering. The

supplier manages the inventories at the retailers by deciding on when and how much to ship to each retailer, and guarantees that neither retailers nor itself will stock-out in any period. The supplier decides on how much to order in each period, and may ship to the retailers immediately or keep inventory for replenishing retailers in later periods. When an order is placed at the supplier in a period, a fixed order cost independent of the size of order and a variable order cost per unit ordered are incurred. For each unit kept in inventory at a retailer or supplier in a period, a holding cost is incurred. The amount of inventory kept at a retailer cannot exceed the storage capacity of that retailer. A fleet of homogeneous vehicles based at the supplier is available for distribution. Each vehicle, departing from and returning back to the supplier, can visit multiple retailers in a multi-stop route without exceeding its capacity. A vehicle traveling from a facility to another facility incurs a transportation cost. We assume that the vehicle can only perform a single tour in every period. The PRP is to decide on when and how much to order at the supplier, when and how much to ship to each retailer, and the routing of vehicles such that the sum of fixed and variable order costs, transportation cost as well as inventory carrying costs at the supplier and retailers is minimized.

We propose an a priori tour-based three phase heuristic (denoted by 3P) for the problem. We solve a giant traveling salesman problem (TSP) over all retailers and supplier in the first phase. In the second phase, we formulate the problem as a mixed integer program (MIP) with an approximation of the true routing cost using the giant tour obtained in the first phase and solve it. Finally, we solve a capacitated vehicle routing problem (CVRP) or a TSP depending on the number of vehicles needed in each period in the third phase so as to obtain a feasible solution.

We have performed computational experiments on benchmark instances introduced by [6]. These instances involve respectively 14, 50, and 100 retailers in three sets of instances. Each set has 480 instances. There is a single vehicle in the instances with  $N = 14$  whereas the number of available vehicles is unlimited for the other instances. We have compared our results with the results of the heuristics (IM-VRP and IM-MultiTSP) proposed by [1], the adaptive large neighborhood search heuris-

Heuristic	$N = 14$		$N = 50$		$N = 100$	
	Ave. Cost	# Best	Ave. Cost	# Best	Ave. Cost	# Best
3P	179325.3	255	587908.2	221	1085460.3	195
IM-VRP	–	–	588182.7	185	1086168.9	206
IM-MultiTSP	179414.5	181	589774.5	48	1090640.8	19
ALNS	181802.9	2	590210.0	24	1089588.6	3
H	180829.9	0	592608.1	6	1092508.9	57

Table 1: Summary of computational results

tic (ALNS) proposed by [2], and the local search heuristic (H) proposed by [6]. We have solved the MIP in the second phase using Cplex 12.5, the CVRPs in the third phase using the heuristic of [7], and the TSPs in the first and third phases using the Concorde solver [5].

A summary of the computational results on the instances introduced by [6] is presented in Table 1, where first column indicates the name of the heuristic, the columns named as 'Ave. Cost' the average of the objective function values obtained over 480 instances, and the columns named as '# Best' the number of the times a heuristic finds the smallest objective function value. Note that columns 2-3, 4-5, 6-7 show the results for the instances with  $N = 14, 50,$  and  $100,$  respectively, and the column 3 shows the number of the times a heuristic finds the optimal solution. The computational results reveal that our heuristic yields very good results improving many existing results. We should note that the computational time 3P required did not exceed six minutes even for the largest instances.

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# New cutting planes for inventory routing

*Pasquale Avella*

*Dipartimento di Ingegneria, Universita del Sannio  
avella@unisannio.it*

*Maurizio Boccia*

*Dipartimento di Ingegneria, Universita del Sannio  
maurizio.boccia@unisannio.it*

*Laurence Wolsey*

*CORE, Université catholique de Louvain  
laurence.wolsey@uclouvain.be*

## Abstract

Recently various authors have worked on a vendor-managed inventory routing problem. Here we present a new way to obtain valid inequalities for this problem and then present several families of inequalities obtained by this approach.

## 1 Introduction

Recently various authors [3, 5, 6, 7, 8, 9] have worked on a vendor-managed inventory routing problem appearing in Archetti et al [2]. Here we present a new way to obtain valid inequalities for this problem and then present several families of inequalities obtained by this approach. Essentially we look at the single period subproblem arising in each period (to obtain the full problem, it suffices to add the standard flow conservation at each client in each period ( $s_{t-1}^i + x_t^i = D_t^i + s_t^i, 0 \leq s_t^i \leq \bar{S}^i$ )).

The talk will also involve proofs of validity, separation heuristics and the latest computational results.

## 2 The single-period IRP subproblem

Let  $I$  denote the set of clients, node 0 the depot and  $I0 = I \cup \{0\}$ . In the single period model presented below,  $C$  is the capacity of each of the  $K$  vehicles and  $W^i$  an upper bound on the amount that a vehicle can deliver to client  $i$  in the period. Each vehicle if used must make a subtour visiting a subset of the clients beginning and ending at the depot. Each client is visited by at most one vehicle.  $D^i$  is the demand of client  $i$  in the given period and  $\bar{S}^i = W^i - D^i$  is then the upper bound on the stock at the end of the period.  $A$  are the arcs of the complete digraph with node set  $I0$ .

We use the variables:

$x^i$  is the amount delivered to client  $i$

$z^0$  is the number of vehicles used in the period

$z^i = 1$  if client  $i$  receives a delivery in the period, and  $z^i = 0$  otherwise

$y^{ij} = 1$  if one of the vehicles travels on the arc  $ij \in A$   $s^i$  denotes the stock at client  $i$  at the end of the period.

The problem treated is:

$$\sum_{i \in I} x^i \leq C z^0, \quad x^i \leq W^i z^i, \quad z^0 \geq z^i, \quad i \in I \quad (1)$$

$$\sum_{j \in I0} y^{ij} = \sum_{j \in I0} y^{ji} = z^i, \quad j \in I0, \quad (2)$$

$$\sum_{i \in I0 \setminus S, j \in S} y^{ij} \geq z^k \quad \forall S \subset I, \quad \forall k \in S. \quad (3)$$

$$C \sum_{i \in I0 \setminus S, j \in S} y^{ij} \geq \sum_{k \in S} x^k \quad \forall S \subset I. \quad (4)$$

$$x, \in R_+^T, z^i \in \{0, 1\} \quad i \in I, z^0 \in \{0, 1, \dots, K\}, y^{ij} \in \{0, 1\}^A \quad (5)$$

Optional:

$$x^i \leq D^i + s^i, 0 \leq s^i \leq W^i - D^i, \quad \forall i \in I \quad (6)$$

$$s^i \leq W^i - D^i, s^i - x^i \leq W^i - 2D^i \quad \forall i \in I \quad (7)$$

where  $W^i \leq C$  for all  $i \in I$ .

## 3 Metric Inequalities

In the following inequalities, we assign non-negative weights  $\mu^{ij}$  to the set of arcs  $A$ .

**Proposition 1.** *Let  $S \subset I$ , and let  $t \in T$ . The Metric Inequality*

$$\sum_{i \in I \cup \{0\}} \sum_{j \in I} \mu^{ij} y^{ij} \geq \sum_{i \in S} x^i \quad (8)$$



is valid for the set (1)-(5) if  $\mu^{ij} \geq 0 \forall (i, j) \in A$  and

$$\sum_{ij \in P} \mu^{ij} \geq \min(C, \sum_{i \in v(P) \cap S} W^i), \quad (9)$$

for each subtour  $P$  starting and ending at the depot, where  $v(P)$  is the set of nodes of  $P \cap I$ .

Here we first show two classes of metric inequalities.

### 3.1 A Simple Metric Inequality

**Proposition 2.** Let  $S \subseteq I$  with  $(S_1, S_2)$  a partition of  $S$ .

$$\sum_{i \in I \setminus S} \sum_{j \in S_1} C y^{ij} + \sum_{i \in I \setminus S} \sum_{j \in S_2} W^j y^{ij} + \sum_{i \in S_2} \sum_{j \in S_2} \min(W^j, C - W^i) y^{ij} + \sum_{i \in S_2} \sum_{j \in S_1} (C - W^i) y^{ij} \geq \sum_{i \in S} x^i \quad (10)$$

is a Metric Inequality with

$$\begin{aligned} \mu^{ij} &= C \text{ for } ij \in (I \setminus S : S_1), \\ \mu^{ij} &= W^j \text{ for } ij \in (I \setminus S : S_2), \\ \mu^{ij} &= \min\{W^j, C - W^i\} \text{ for } ij \in (S_2 : S_2), \\ \mu^{ij} &= C - W^i \text{ for } ij \in (S_2 : S_1), \\ \mu^{ij} &= 0 \text{ otherwise.} \end{aligned}$$

**Example 1.**  $C = 254$ .  $W = \{195, 254, 150, 129, 128\}$ ,

Taking  $S = \{2, 5\}$ ,  $S_2 = \{5\}$  and  $I \setminus S = \{0, 1, 3, 4\}$ , the simple metric inequality is:

$$254(y^{02} + y^{12} + y^{32} + y^{42}) + 128(y^{05} + y^{15} + y^{35} + y^{45}) + 126y^{52} \geq x^2 + x^5.$$

**Proposition 3.** Let  $S \subseteq I$  be a subset of the customers. The Generalized Inequality:

$$\sum_{i \in I} \sum_{j \in S} L_j y^{ij} + \sum_{i \in S} \sum_{j \in S} R_{ij} y^{ij} \geq \sum_{i \in S} x^i \quad (11)$$

is valid for IRP if:

- i)  $W^i \leq L_i \leq C$  for each  $i \in S$ ;
- ii)  $L_i + R_{ij} \geq \min(W^i + W^j, C)$  for each  $ij \in (S : S)$ ;
- iii)  $L_i + R_{ij} + R_{jk} \geq C$  for each  $ij, jk \in (S : S)$ .
- iv)  $R_{ij} \geq 0$  for all  $ij \in (S : S)$ .

Note that these conditions can be extended to deal with paths with three or more clients.

## 4 Extensions

Proposition 1 can be easily generalized, allowing us to obtain inequalities also involving stock variables. Also several multi-period variants of the metric inequalities can be described.

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# The Dynamic Lot-sizing Problem with Convex Economic Production Costs and Setups

*Kian, Ramez*  
*Bilkent University*  
*ramezk@bilkent.edu.tr*

*Gurler, Ulku*  
*Bilkent University*  
*ulku@bilkent.edu.tr*

*GuBerk, Emre*  
*Bilkent University*  
*eberk@bilkent.edu.tr*

## Abstract

In this extended abstract we review the highlights of our recently published paper [1] which considers the uncapacitated lot-sizing problem with deterministic demands. Production costs consist of convex costs that arise from economic production functions plus set-up costs. A mixed integer, non-linear programming formulation together with structural results are provided which are used to construct a forward dynamic-programming algorithm that obtains the optimal solution in polynomial time. For positive setup costs, the generic approaches are found to be prohibitively time-consuming; therefore the forward DP algorithm is modified via the conjunctive use of three rules for solution generation. Additionally, we propose six heuristics. Two of these are single-step Silver-Meal (H1) and EOQ (H2) heuristics for the classical lot-sizing problem. The third is a variant of the Wagner-Whitin (H3) algorithm. The remaining three heuristics are two-step hybrids that improve on the initial solutions of the first three by exploiting the structural properties of optimal production subplans. An extensive numerical study has been done to test the heuristics.

## 1 Introduction

In this paper, we consider the problem of dynamic lot-sizing in the presence of polynomial-type convex production functions and non-zero setup costs. The dynamic

lot-sizing problem is defined as the determination of the production plan that minimizes the total (fixed setup, holding and variable production) costs incurred over the planning horizon for a single, storable item facing deterministic demands. The so-called classical dynamic lot-sizing problem was first analyzed by [2]. They established that, in an optimal plan with positive fixed setup costs and linear production and holding costs, production is done in a period only if its net demand (actual demand less inventories) is positive, and a period's demand is satisfied entirely by production in a single period (that is, integrality of demand is preserved.) For details on such results, we refer the reader to the reviews in [3], [6], [7], [4] and [8].

Most of the existing works on the dynamic lot-sizing problem deal with linear and/or concave production functions rather than convex functions. Our work differs from the existing literature in our main assumption about the structure of production costs. Specifically, we consider variable production costs in period  $t$  of the polynomial form  $\sum_{n=1}^m w_t^n q_t^{r_t^n}$  where  $q_t$  denotes the quantity produced in the period,  $w_t^n$  and  $r_t^n$  are positive constants and  $m$  is the number of resources. The assumed non-linearity are originated from a number of industrial settings as discussed in [1]: From the input-output relation which follows a Cobb-Douglas or Leontieff production function or from the penalties of the ecological factors as carbon emission fines as in [5].

## 2 Problem Formulation

Parameters and variables:

- $q_t$  amount of the production in period  $t$ .
- $y_t$ : binary variable for detecting setup in period  $t$ .
- $K_t$  fixed setup cost in period  $t$ .
- production cost:  $\sum_{j=1}^m w_t^j q_t^{r_t^j}$ , where  $w_t^j$  and  $r_t^j$  are non-negative constants. We assume that  $r_t^j > 1 \quad \forall j, t$  resulting in convex variable production costs.
- $h_t$  unit holding cost at period  $t$ .
- $D_{u,v} = d_u + d_{u+1} + \dots + d_v$  denote the total demand of periods  $\{u, u+1, \dots, v\}$
- $P_{u,v}$  denote the production planning sub problem of periods  $\{u, u+1, \dots, v\}$

We formulate the problem as a MINLP problem. This allows us to establish certain structural properties of the optimal solution. We can state problem  $P_{u,v}$  formally as follows.

$$\min_{q_u, \dots, q_v} F_{u,v} = \sum_{t=u}^v \left[ (K_t y_t + \sum_{j=1}^m w_t^j q_t^{*j} + \left( \sum_{i=t}^v h_i \right) q_t) - \sum_{t=u}^v h_t D_{u,t} \right] \quad (1)$$

s.t.

$$\sum_{i=u}^t q_i \geq D_{u,t} \quad t \in \{u, \dots, v\} \quad (2)$$

$$q_t \geq 0, \quad t \in \{u, \dots, v\} \quad (3)$$

$$q_t \leq D_{t,v} y_t, \quad t \in \{u, \dots, v\} \quad (4)$$

$$y_t \in \{0, 1\}, \quad t \in \{u, \dots, v\} \quad (5)$$

The nonlinear convex production costs are the key difference between our model and the classical well-known model introduced by [2] is that the fundamental zero inventory property (ZIP) of the optimal solution in the classical model does not necessarily hold. To illustrate this point, consider  $P_{1,T}$  for the following simple example. For  $T = 2$ ,  $K_t = K = 700$ ,  $h_t = h = 1$ ,  $m = 1$ ,  $w_t^1 = w = 0.01$ ,  $r_t^1 = r = 2$  for  $1 \leq t \leq T$  and  $\mathbf{d} = (100, 300)$ . The optimal plan for this problem gives  $q_1^* = 175$  and  $q_2^* = 225$  and  $I_1^* \times q_2^* \neq 0$ .

### 3 Analytical Result and Solution Approaches

**Definition 1** In a given production plan,  $Q_{ij}$  for periods  $\{i, \dots, j\}$ ,

- (1) period  $t$  is a regeneration point if  $I_{t-1} = 0$ ;
- (2) a sequence of periods  $\{u, u+1, \dots, v\}$ , for  $i \leq u \leq v \leq j$ , is a generation, denoted by  $\langle u, v \rangle$ , if  $I_{u-1} = I_v = 0$  and  $I_t > 0$  for  $t \in \{u, u+1, \dots, v-1\}$ ;
- (3) the production plan of a generation is called a production sequence.

**Theorem 1** (Optimal Production Plan Structure I) In an optimal production plan  $Q_{1,T}^*$ , for any generation  $\langle u, v \rangle$ ,

- (i)  $Q_{uv}^* = (d_u)$  if  $1 \leq u = v \leq T$ ,
- (ii)  $Q_{u,v}^* = (D_{uv}, 0, \dots, 0)$  if  $1 \leq u < v \leq T$  and  $r_t^n \leq 1$  for  $t \in [u, v]$ ,
- (iii)  $Q_{u,v}^* = (q_u^*, \dots, q_v^*)$  is of class  $G$  if  $1 \leq u < v \leq T$  and  $r_t^n > 1$  for  $t \in [u, v]$ ,

**Theorem 2** (Optimal Production Plan Structure II) *If  $r_t^n \geq 1$  for all  $n, t$ , in an optimal production plan, for generations  $\langle u, v \rangle$  and  $\langle v + 1, v' \rangle$ ,*

$$\sum_{n=1}^m r_{v+1}^n w_{v+1}^n (q_{v+1}^*)^{r_{v+1}^n - 1} \leq \sum_{n=1}^m r_t^n w_t^n q_t^{*r_t^n - 1} + \sum_{i=l}^v h_i, \quad (6)$$

**Lemma 1** *If  $r_t^n \geq 1$  and  $K_t = 0 \quad \forall t$ , in an optimal production plan, for generation  $\langle u, v \rangle$ ,  $q_j^* > 0$  if  $q_t^* > 0$  for  $u \leq t < j \leq v$ .*

### 3.1 Dynamic programming formulations

*Backward formulation:*

$$J_{t-1}^T(I_{t-1}) = \min_{q_t \geq \max(0, d_t - I_{t-1})} \left\{ K_t \mathbb{1}_{q_t > 0} + h_t I_t + \sum_{n=1}^m w_t^n q_t^{r_t^n} + J_t^T(I_t) \right\} \quad t \in \{1, \dots, T\} \quad (7)$$

*Forward formulation:*

$$f_t^* = \min_{1 \leq i \leq t-1} \{f_{i-1}^* + g_{i,t}\}, \quad (8)$$

where  $g_{i,t}$  is the cost of a type-G subplan within  $[i, t]$ . For the detail of the heuristics and numeric test bed the reader may refer to [1]. However, a compact statistics of the heuristics performances is provided in Table 1.

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Demand	Algorithm	min	max	median	average	N(0)	N(-)
d1	<i>H1</i>	1.23	89.23	17.97	24.43	0	0
	<i>H2</i>	0.08	65.03	6.24	12.47	0	0
	<i>H3</i>	0	65.03	5.01	12.76	124	0
d2	<i>H1</i>	0.79	28.2	10.72	11	0	0
	<i>H2</i>	0	19.9	3.24	4.71	8	0
	<i>H3</i>	0	19.9	1.58	4.16	161	0
d3	<i>H1</i>	0.5	7.59	3.18	3.33	0	0
	<i>H2</i>	0	7.4	1.69	1.99	36	0
	<i>H3</i>	0	5.37	0.44	1.04	235	0
d1	<i>H4</i>	1.09	44.17	13.06	15.39	0	0
	<i>H5</i>	0.06	18.87	1.8	3.36	0	0
	<i>H6</i>	-0.01	18.87	1.73	3.68	157	2
d2	<i>H4</i>	0.79	15.84	6.78	6.63	0	0
	<i>H5</i>	-0.01	2.28	0.29	0.4	65	5
	<i>H6</i>	-0.01	2.37	0.19	0.34	223	3
d3	<i>H4</i>	0	7.04	1.37	1.94	114	0
	<i>H5</i>	0	1.29	0	0.17	636	0
	<i>H6</i>	0	0.65	0	0.02	808	0

Table 1: Percentage deviation statistics for heuristics *H1* - *H6*.

# Towards a Universal Solver for Lot Sizing Problems

*Rui Rei*<sup>1</sup>

*INESC TEC and Faculdade de Ciências, Universidade do Porto, Portugal*  
*rui.rei@dcc.fc.up.pt*

*João Pedro Pedroso*

*INESC TEC and Faculdade de Ciências, Universidade do Porto, Portugal*  
*jpp@fc.up.pt*

## Abstract

Lot sizing is a hard problem with many variants including more or less practical details. We depart from a standard formulation for the base problem in mixed-integer programming, and describe a general framework that can be applied to obtain good solutions for more complex variants of the problem, such as lot sizing and scheduling.

## 1 Introduction

The lot sizing problem consists of determining the optimal production plan for a set of products under a given planning horizon, and has been proven to be NP-hard in most practical situations [1]. In this work we study the so-called *big bucket* model, under which multiple products may be produced in each planning period. A base formulation for the problem, in mixed-integer linear programming (MIP), is shown below.

$$\text{minimize } \sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} (f_t^p y_t^p + c_t^p x_t^p + h_t^p I_t^p) \tag{1}$$

$$I_{t-1}^p + x_t^p - I_t^p = d_t^p \quad \forall p \in \mathcal{P}, t \in \mathcal{T} \tag{2}$$

$$\sum_{p \in \mathcal{P}} x_t^p + \sum_{p \in \mathcal{P}} g_t^p y_t^p \leq M_t \quad \forall t \in \mathcal{T} \tag{3}$$

$$x_t^p \leq (M_t - g_t^p) y_t^p \quad \forall p \in \mathcal{P}, t \in \mathcal{T} \tag{4}$$

$$I_0^p = 0 \quad \forall p \in \mathcal{P} \tag{5}$$

$$x_t^p, I_t^p \geq 0 \quad \forall p \in \mathcal{P}, t \in \mathcal{T} \tag{6}$$

$$y_t^p \in \{0, 1\} \quad \forall p \in \mathcal{P}, t \in \mathcal{T} \tag{7}$$

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<sup>1</sup>Funded by PhD grant SFRH/BD/66075/2009 from the Portuguese Foundation for Science and Technology (FCT).



For period  $t$  and product  $p$  we have the following decision variables:  $x_t^p$ , determining the quantity of product  $p$  that is produced in period  $t$ ;  $y_t^p$  (binary) is 1 if product  $p$  is to be produced in period  $t$ , 0 otherwise;  $I_t^p$  is the amount of product  $p$  in inventory at the end of period  $t$ . Problem data includes:  $f_t^p$ ,  $c_t^p$ ,  $h_t^p$ , the cost of setup, production, and inventory (respectively);  $d_t^p$  is the demand for product  $p$  at period  $t$ ;  $g_t^p$  is the setup time required for producing  $p$  at period  $t$ ; and  $M_t$  is the production time available in period  $t$ .

For this basic formulation, solutions of very good quality can be found by state-of-the-art MIP solvers for considerably large and difficult instances. However, results degrade significantly when additional practical aspects are incorporated into the model (*e.g.*, backlogging, multiple machines, scheduling with sequence-dependent setup times and costs). These additions make the model progressively harder to solve exactly in reasonable time and, in some cases (*e.g.*, scheduling), the solutions reached in limited time may be of poor quality.

We propose a general framework for tackling these more intricate variants of lot sizing, based on the combination of Monte-Carlo tree search with linear programming. The framework aims to be general and powerful enough to attain good quality solutions in reasonable time for large lot sizing instances.

## 2 Monte-Carlo Tree Search

Monte-Carlo tree search [2] (MCTS) is an iterative procedure in which a search tree is asymmetrically constructed, attempting to expand the tree towards its most promising parts while balancing exploitation of known good branches with exploration of seemingly unrewarding branches. The algorithm is based on the idea of Monte-Carlo evaluation, *i.e.*, the reward associated with a particular internal node—corresponding to a partial solution—can be estimated from the results of randomized simulations started from that node. Each node keeps track of the number of simulations run from its state as well as their outcomes, and these data are used to produce an estimate value for the node when deciding how to expand the tree. Each iteration of MCTS can be divided into the following four steps:

1. **Selection:** starting from the root node, select the child node which currently looks more *promising*, according to a score function  $U(n)$ . This is done recursively until a node  $n$  which has not yet been expanded is reached.
2. **Expansion:** the children of the selected node  $n$  are created by applying possible decisions in  $n$  to copies of  $n$ .
3. **Simulation:** from each node created in the expansion step, perform a simulation until a terminal state (*i.e.*, a complete solution or infeasibility) is reached, and record its outcome.

4. **Backpropagation:** propagate the outcome of each simulation up the tree until the root node is reached; this updates statistics (simulation scores and counts) on all nodes between the selected node and the root.

The steps above, adapted for the the problem of combinatorial optimization, are summarized in the pseudo-code in Algorithm 1.

---

**Algorithm 1:** MCTS for (minimization) combinatorial optimization.

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**Data:** problem instance  $I$   
**Result:** upper bound on optimal value (minimization problems)

- 1  $r \leftarrow$  create root node with initial state from  $I$
- 2  $z^* \leftarrow \infty$
- 3 **repeat**
- 4      $n \leftarrow$  starting from  $r$ , recursively select child node with maximum  $U(n)$
- 5      $C \leftarrow$  set of child nodes obtained by expanding  $n$
- 6     **foreach**  $c \in C$  **do**
- 7         run a simulation from  $c$
- 8          $z \leftarrow$  result of the simulation
- 9         propagate  $z$  up until reaching  $r$
- 10        **if**  $z < z^*$  **then**
- 11             $z^* \leftarrow z$
- 12 **until** computational budget is depleted
- 13 **return**  $z^*$

---

### 3 Lot sizing and scheduling

The MCTS framework can be used to tackle the problem of integrated lot sizing and scheduling, considering sequence-dependent changeovers. This problem, tackled *e.g.*, with pattern generation for the scheduling part in [4], is known to be difficult to solve using general-purpose MIP solvers, as the number of integer variables is dramatically increased.

We propose a framework where the tree search will instantiate the integer variables of model (1)–(7), based on information provided initially by the linear relaxation of this incomplete model. Leaves of this tree correspond to having all the setup variables instantiated; this will allow finding appropriate schedules in each period, which in turn allows the evaluation of the objective function. This value—implied by the setup variables but missing in the base formulation—will provide the main guidance for MCTS, reinforcing search in nodes that have provided solutions, but

have not yet been sufficiently explored. As an alternative, nodes can be evaluated using solutions obtained through general-purpose MIP heuristics such as relax-and-fix [5, 6] or the feasibility pump heuristic [3]. Yet another alternative, which is computationally cheaper, is to make a probabilistic rounding of the binary variables based on the linear relaxation solved at the node.

Besides simulation, uncertainty can also be introduced in the selection step of MCTS. Here, the greedy selection rule can be replaced by a probabilistic choice (assigning to each node a probability proportional to its current score) as a way to prevent the search from stalling.

## 4 Remarks

We propose a general framework for tackling hard lot sizing problems, based on the idea of partitioning model variables into two sets: one concerning important decisions, consisting of the binary setup variables, which are fixed in the enumeration (tree search) stage; and the other consisting of the production and sequencing variables, for which values are determined after fixing all variables in the first group.

This framework allows for the inclusion of other details which cannot be represented in a MIP formulation, such as non-linear objective functions, or the interdependence between multiple competing agents in a more detailed market representation.

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# Two neighborhood search approaches for the dynamic capacitated lotsizing problem with scarce setup resources

*Düerkop, Sascha*

*University of Cologne, Dep. for Supply Chain Management and Production,  
Albertus-Magnus-Platz, D-50932 Cologne, Germany  
dueerkop@wiso.uni-koeln.de*

*Briskorn, Dirk*

*Schumpeter School of Business and Economics Wuppertal, Dep. for Production and  
Logistics, Rainer-Gruenter-Str. 21, D-42119 Wuppertal, Germany  
briskorn@uni-wuppertal.de*

*Tempelmeier, Horst*

*University of Cologne, Dep. for Supply Chain Management and Production,  
Albertus-Magnus-Platz, D-50932 Cologne, Germany  
tempelmeier@wiso.uni-koeln.de*

## Abstract

We present a model for a capacitated lotsizing problem with an integrated coordination of a common setup resource. Furthermore we present a problem specific neighborhood search algorithm to solve the problem efficiently. We close the presentation with an outlook and an overview of the positioning of known approaches in terms of complexity and size of neighborhood.

## 1 Introduction

In recent years a common setup operator for a dynamic lotsizing problem was found in a number of different practical cases (see for example [1] and [2]). The common setup resource in most cases is a setup operator that carries out all setup operations on all machines. The practical cases all have in common that setup states can be preserved, setup costs as well as times are sequence dependent and products are pre-assigned to a single machine. Neglecting the setup operator the problem decomposes into problems concerning a single machine each. Given a common setup operator this solution approach is not suitable anymore as it might cause conflicts in the schedule

of the setup operator. Taking a common setup resource into account a simultaneous lotsizing and scheduling of the setup operator is required.

This problem was formulated as a mixed integer program and solved by a Fix-and-Optimize (see for example [3]) approach in [1] initially. The Fix-and-Optimize heuristic is a particular neighborhood search that is well-known to perform good even on large instances of the Capacitated Lotsizing Problem (CLSP) and its variations. Nevertheless, this approach is not specifically hand-tailored for the problem extension of a common setup resource. For that reason our work focuses on building a new neighborhood search that focuses on the common setup resource.

## 2 Problem description

The problem has the following characteristics:

- Simultaneous lotsizing and scheduling
- Big-bucket model
- Products can be produced only once per macro-period
- Setups are carried out by a common setup operator
- The setup operator can only carry out a single setup at a time
- Each product is uniquely assigned to a single machine
- Each machine has a limited capacity
- Setup times and costs are sequence dependent
- On each machine the final setup state in each period can be carried over to the next
- Backlogging and overtime are not allowed
- The objective is to minimize the overall costs (sum of holding and setup costs)

## 3 Solution approach

The basis of the solution approach is the MIP formulation in [1]. To solve the above described problem by a more specific approach we designed two different neighborhood searches. In the first neighborhood search approach, the remaining subproblem,

which has to be solved for each member of the search space, can be solved in polynomial time. In the second neighborhood search approach the remaining subproblem is again a MIP.

Both search algorithms are implemented in *C++* while the remaining subproblems are solved with *CPLEX*.

We compare our neighborhood searches with the approaches of [1] in terms of complexity of the subproblem and size of search space. This trade-off and the positioning of the different approaches within this tradeoff give an impression of the long-term goals of our research.

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# On robust vs. flexible dynamic lot sizing subject to capacity constraints and random yield

*Stefan Helber*  
*Leibniz Universität Hannover*  
*stefan.helber@prod.uni-hannover.de*

*Florian Sahling*  
*Leibniz Universität Hannover*  
*florian.sahling@prod.uni-hannover.de*

*Katja Schimmelpfeng*  
*Universität Hohenheim*  
*katja.schimmelpfeng@uni-hohenheim.de*

## Abstract

We study the problem to determine time-phased production quantities in a single-stage, multi-period and multi-product setting. The problem is a generalization of the well-known deterministic Capacitated Lot Sizing Problem (CLSP). While the CLSP does not consider quality imperfections, we assume that the production process is characterized by substantial yield uncertainty described via an interrupted geometric distribution. The idea is that during the production of each single product the process goes “out of control” with a specific probability so that this and all following product units of the particular lot have to be scrapped. In this situation, the lot size has a paramount effect on the average cost of non-defective product units and lot sizing becomes immensely important. Furthermore, a decision has to be made whether to operate with a static robust production plan or to use some flexible production policy.

## 1 Introduction

We consider the problem to determine lot sizes for multiple products in a multi-period setting with deterministic and time-dependent demand. The different products require the same production resource, i.e., a machine, which has a limited period capacity that can be extended via overtime. In order to be able to produce, the machine requires product-specific setup operations, so that a lot sizing problem arises. From this point of view, the problem resembles the well-established Capacitated Lot

Sizing Problem (CLSP). However, as opposed to the CLSP, an additional difficulty stems from yield uncertainty. We consider the case that during the production of each single product unit the process goes “out of control” with a specific probability so that this and all following product units of the particular lot have to be scrapped. In this situation, the lot size has a paramount effect on the average cost of non-defective product units and lot sizing becomes immensely important. In addition, as the yield of each production lot is random, the end-of-period inventory is also a random quantity. In such a situation, it may occasionally be impossible to meet the customer demand without delay. Service level measures can be used to quantify the service to the customers that results from any given production schedule.

We present an approximate modeling approach to determine robust production plans subject to this yield uncertainty. To this end, we approximate the expected end-of-period values of physical inventory and backlog and minimize the expected cost subject to a backlog-oriented  $\delta$ -service level constraint. This robust plan can be used as component of a heuristic scheduling policy subject to yield uncertainty. Both approaches can be compared to an exact stochastic dynamic programming approach, which we also describe briefly.

## 2 Problem setting and related literature

Consider the following production situation: In order to produce different types of products, a single production resource, e.g., a machine, is used. This machine has a limited production capacity. In order to be able to produce a specific product, the machine requires a setup operation. After the setup, multiple product units are sequentially produced within a production lot, i.e., a serial production batch. The production process can be either “in control” (and lead to “good” parts that conform to the product specification) or “out of control” (and lead to defective parts that have to be scrapped). In Figure 1 we show an example in which three different lots of size seven are produced. During the production of the first lot, the process goes out of control while attempting to produce product unit 5. The same happens during the production of product unit 3 of the third lot. However, during the production of the second lot, the process stays in control and the yield equals the lot size. Such phenomena occur, for example, if tools break, pipes congest, etc.

We assume that after the setup operation, the process is always (initially) in control. It stays in control with probability  $p$  during each processing operation for an individual product unit which belongs to the currently processed lot. In this situation, the random yield  $Y$ , i.e., the number of good parts in a lot of size  $q$  follows an Interrupted Geometric (IG) distribution with parameters  $p$  and  $q$ . This distribution is defined as follows:



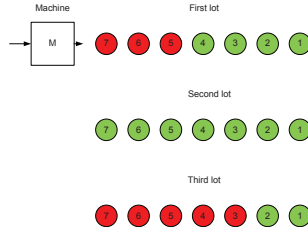


Figure 1: Example of three different production runs with different yield

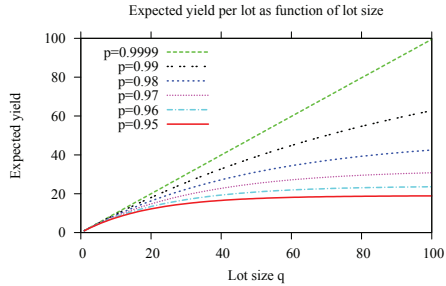


Figure 2: Expected yield functions

$$\text{Prob}[Y = k] = \begin{cases} p^k \cdot (1 - p) & k = 0, 1, \dots, q - 1 \\ p^q & k = q \end{cases} \quad (1)$$

If the yield  $Y$  of a lot of size  $q$  is a random quantity and follows the IG distribution, the expected value of conforming parts

$$E[Y] = \frac{p(1 - p^q)}{1 - p} \quad (2)$$

increases with the lot size  $q$ , but the marginal increase decreases with  $q$  as shown in the examples of Figure 2 for a success parameter  $p = 0.8$ . For large lot sizes, a small decrease of the success probability  $p$  leads to a remarkable fraction of bad parts.

We follow the conceptual approach that was presented for the stochastic CLSP with random demand in [2]. Similar approaches to determine robust production plans subject to demand uncertainty were presented in [6] and [5], while production policies

for the case of both demand and yield uncertainty were recently discussed by [4]. Lot sizing policies for interrupted geometric yield are discussed in [7], [1], and [3].

### 3 Robust planning vs. flexible scheduling

In our approach we assume there will be demand for the products after the end of the current planning horizon. We compare three different approaches to deal with yield uncertainty. We assume that in each approach the lot size is used which minimizes long-term average costs. In a robust planning approach, the number of those lots per product and periods is determined such that a pre-specified  $\delta$ -service level is met. In a semi-flexible approach, the number of lots is heuristically adjusted to the yield outcome. Finally, for very small examples, the optimal policy can be determined via backward induction in a stochastic dynamic program.

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# Stochastic capacitated lot sizing in closed-loop supply chains

*Timo Hilger*  
*University of Cologne*  
*timo.hilger@wiso.uni-koeln.de*

*Florian Sahling*  
*Leibniz University Hannover*  
*sahling@prod.uni-hannover.de*

*Horst Tempelmeier*  
*University of Cologne*  
*tempelmeier@wiso.uni-koeln.de*

## Abstract

We present a stochastic version of the single-level, multi-product capacitated lot sizing problem with remanufacturing (SCLSP-RM) on separate resources. The objective is to determine robust (re)manufacturing plans for uncertain demand and returns in order to minimize the expected costs with subject to service-level constraints. We propose a non-linear model formulation that is approximated by a scenario approach and a piecewise linear approximation model. In the case of the scenario approach, the expected values of random variables are replaced by sample averages. The idea of the piecewise linear approximation model is to replace non-linear functions by piecewise linear functions. The resulting mixed-integer linear programs can be solved in order to determine robust (re)manufacturing plans.

## 1 Introduction

Closed-loop supply chains are characterized by a forward-oriented material flow of products from the manufacturer to the customer and a backward-oriented material-flow of product returns from the customer to the manufacturer at the product's end-of-use. These product returns can be remanufactured to become as good as new.

## 2 Problem Statement

The objective of the Stochastic Capacitated Lot Sizing Problem with Remanufacturing (SCLSP-RM) is to determine a (re)manufacturing plan for  $K$  products which

minimizes the sum of expected (re)manufacturing, holding and setup costs over a planning horizon of  $T$  periods. Products can be (re)manufactured on a production system, which contains two separated capacity-restricted resources: On the first one, products can only be manufactured, while remanufacturing is located on the second one. For this reason, (separate) setup operations are required for producing and remanufacturing. The expected demands  $E\{D_{kt}\}$  and returns  $E\{R_{kt}\}$ , as well as the variances  $VAR\{D_{kt}\}$  and  $VAR\{R_{kt}\}$  are known as a result of the applied forecasting system. The demand can be satisfied by manufacturing quantities  $q_{kt}$  or remanufacturing quantities  $q_{kt}^r$ . Remanufacturing can only be processed completely, if a sufficient amount of product returns  $R_{kt}$  are available and stored as recoverables inventory  $I_{kt}^{p,r}$ . Contrary, the source of material for the manufacturings is assumed to be unlimited. For the non-linear model formulation of the SCLSP-RM it is referred to [2]. The supply chain of the described (re)manufacturing process is illustrated in Figure 1.

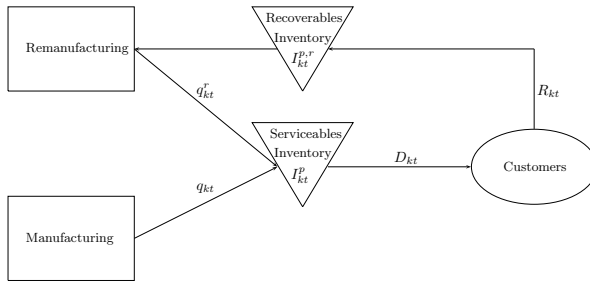


Figure 1: Supply chain with product returns and (re)manufacturing

As a consequence of uncertain period demands and product returns, the net inventory levels of the serviceables and the recoverables are modeled as random variables.

A typical development of the inventory level for the recoverables within the planning horizon is illustrated in Figure 2. The incoming product returns  $R_{kt}$  raise the physical inventory level of the recoverables  $I_{kt}^{p,r}$ . As the recoverables are the source of material for the remanufacturing process, the remanufacturing quantity  $q_{kt}^r$  reduces the inventory level.

If a planned remanufacturing quantity exceeds the recoverables inventory,  $q_{kt}^r > I_{kt}^{p,r}$ , lost-remanufacturings  $LR_{kt}$  have to be taken into account, as they cannot be backlogged and remanufactured in subsequent periods. Consequently, the customer demand cannot be served and the manufacturer is not able to fulfill the predefined  $\delta$  service-level (see [1]) and therefore lost-remanufacturings are restricted in order to generate robust production plans.

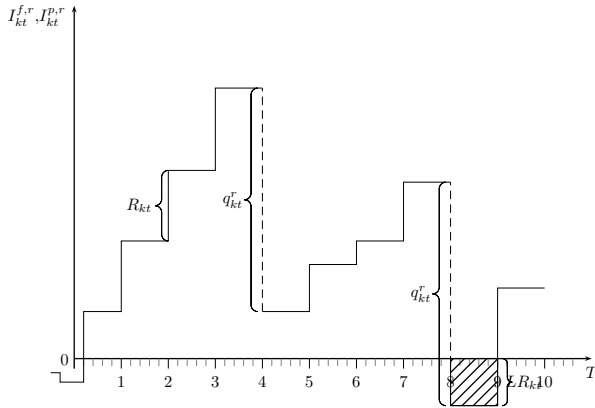


Figure 2: Development of the inventory level of the recoverables

### 3 Numerical Results

Table 1 presents the results of the 1296 test instances, that have been solved both for the piecewise linear approximation model (LIN) and the scenario model (SCN). In order to test the accuracy of the approximation models, the service-levels are verified with a simulation study. Table 1 summarizes the relative and absolute deviation of both of the approximation models from the simulation for the following parameters:

- **SL:** Analyzes the problem instances for which the required service-level is met for all products in the last period of the planning horizon, or for which the service-level is missed by at most 0.1, 0.5 or 1.0 percentage points, respectively.
- **SLP:** Reports the number of products within the test instances for which the required service-level in the last period of the planning horizon is met, or for which the service-level is missed by at most 0.1, 0.5 or 1.0 percentage points, respectively.

It is remarkable that 100.00% of the test instances in the category SL and SLP of both approximation models fulfill the predefined  $\delta$  service-level with an accuracy of one percentage point. If the range of fluctuation is reduced, the results clearly indicate that the piecewise linear approximation model is more accurate than the scenario approach.

The expected deviation in terms of the  $\delta$  service-level is 0.03% (LIN) and -0.08% (SCN). Consequently, in average the piecewise linear approximation model overesti-

Categories	LIN		SCN	
	absolute	percentage	absolute	percentage
SL	1953	75.35%	0	0.00%
SL - 0.1%	2592	100.00%	199	7.68%
SL - 0.5%	2592	100.00%	2274	87.73%
SL - 1.0%	2592	100.00%	2592	100.00%
SLP	37899	97.48%	10703	27.53%
SLP - 0.1%	38880	100.00%	23327	60.00%
SLP - 0.5%	38880	100.00%	38429	98.84%
SLP - 1.0%	38880	100.00%	38880	100.00%
Expected deviation	0.00032	0.03%	-0.00074	-0.08%
Max. expected deviation	0.00144	0.15%	-0.00853	-0.86%
Expected deviation (underestimation)	0.00007	0.01%	0.00145	0.15%
Max. expected deviation (underestimation)	0.00043	0.05%	0.00853	0.86%

 Table 1: Deviation of the numerical results in terms of  $\delta$  service-level

mates the service-level slightly, whereby the scenario approach underestimates it. In terms of the robustness of the production plans, an overestimation is preferable. It indicates that the customers' demand can be served according to the agreed conditions. Nevertheless, the underestimation of the scenario approach is negligible. In general, the variance of the sampling approach leads to more fluctuation of the realized customer service-level than the piecewise linear model. This observation is supported by the results of the maximum expected deviation of the underestimations. If the service-level is underestimated, the scenario model underestimates the service-level in average at a maximum of 0.05%, whereby the average maximum underestimation of the scenario approach is 0.86%. Consequently, even if the piecewise linear approximation model is more accurate than the scenario approach, both model formulations lead to robust and feasible results. Thus, both models accurately model the assumed processes of production and remanufacturing under uncertain demand and product returns.

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# An integrated lot sizing and vehicle routing model for fast deteriorating products

*Tom Vogel*  
*Christian Almeder*  
*European University Viadrina Frankfurt (Oder)*  
*twogel@europa-uni.de*  
*almeder@europa-uni.de*

*Pamela C. Nolz*  
*AIT Austrian Institute of Technology*  
*pamela.nolz@ait.ac.at*

## Abstract

We propose a mixed-integer program for planning the production and distribution of products for a medium-sized bakery company consisting of a single production location and several shops where products are sold. The production planning is based on the general lot sizing and scheduling (GLSP) model and the distribution part is modeled as an inventory routing problem (IRP). We consider the product freshness and the production and distribution costs explicitly in a multi-objective framework. Due to the complexity of the model we solve the problem approximately. The computed approximation of the Pareto front indicates that a considerable improvement of the freshness of products can be achieved with only a small increase of the cost.

## 1 Introduction

Companies producing and selling fast deteriorating goods, such as bakery products, diaries, or catering products are facing the problem that production and distribution of the goods have to be executed fast and storing possibilities are consequently very limited. So it is obvious that an integrated planning approach, where both functions are planned simultaneously, might be even more beneficial than in cases with non-perishable goods.

A typical mid-sized bakery company which we consider as a base for developing and testing such a planning approach consists of a production plant, where the bakery products are produced and geographically dispersed shops which have to be supplied with fresh goods. Furthermore those types of companies are usually facing a harsh

competition on the market. So in order to stay competitive, the companies have to focus on reducing cost while simultaneously providing fresh products to the customer. In the case of bakery goods we are facing for some of them a very fast deterioration which leads to a noticeable quality reduction within a few hours.

We face two alternatives of considering perishability in the planning process. First, a maximum age is determined which has to be respected when establishing a production and distribution plan. This allows to derive cost-minimal plans for a given freshness requirement. Second, the freshness can be an objective to be maximized. This might lead to plans with high cost due to small production and transportation batches. Hence, we choose to investigate this trade-off between cost and freshness through a bi-objective model, which allows us to estimate the Pareto curve and derive managerial implications.

## 2 Problem Description

We consider a mid-sized bakery producing bakery goods which have to be delivered to several shops. Some of the products have a shelf-life of only a few hours, which means that their freshness decreases over time and that they must not be sold after reaching a certain age. On the other hand, costs comprising production and delivery costs are to be minimized.

The production process is captured by a General Lot Sizing Problem (GLSP) (see [2]) which we combine with aspects of an Inventory Routing Problem (IRP). The GLSP aims to integrate lot sizing and scheduling of several products on a single capacitated machine, where the schedule is independent of predefined time periods. The IRP deals with the distribution (or collection) of a product to (from) several facilities over a given planning horizon (see [1]). Basically, three different decisions are addressed: when to visit each facility, how much to distribute/collect each time a facility is served and how to route the vehicle(s). The general objective of an IRP is to minimize routing and inventory costs.

We face several trade-offs in our integrated model. First of all, the general trade-off between our two objectives optimizing costs and freshness of the products. Producing and delivering all products early in the morning might lead to low costs. However, the earlier goods are delivered, the less fresh they are when sold to the customers. Second, a trade-off situation with low routing costs but high production costs or the opposite has to be faced. Distribution can be optimized by combining on one route the delivery to bakeries which are located close-by. However, this possibly implies that the production schedule needs to be adjusted to the optimized delivery routes and therefore causes additional costs.



### 3 Solution Approaches

We solve our problem with three different solution approaches, considering always only one of the two objective functions. First, we try to solve the integrated model exactly with the help of CPLEX.

For the purpose of comparing our global integrated production-distribution model with the traditional sequential approach we mimic this method by modifying the integrated model. In order to solve the production part alone we remove all distribution-related decision variables and constraints. Solving the resulting modified GLSP-like problem delivers the solution for the production part. Then all the production related variables are fixed to their computed values, and the remaining distribution decisions are calculated by solving the routing model.

Third, we developed an iterative relax&fix- and fix&optimize heuristic. In the first part the routing is relaxed, the resulting model is solved, and the binary decision variables (i.e., setup and production date) are fixed. To get a feasible solution, the model is now solved by setting the routing variable to binary (instead of relaxing). After that, the iterative fix&optimize procedure starts fixing/freeing either the binary tour variables or the setup and production variables and solving the resulting model.

### 4 Computational Results

For the test purpose we generated 81 test instances. The instances differ concerning the number of products, the number of bakeries, the number of vehicles, and the vehicle capacity. For each of the four parameters three different settings were chosen, subsequently we get  $3^4 = 81$  instances.

We utilize CPLEX Optimization Studio 12.5 with a maximum runtime of 6 hours. We get the results presented in Table 1 by minimizing costs. We test two different settings regarding the MIP Emphasis of CPLEX, "Emphasize optimality over feasibility" and "Emphasize feasibility over optimality".

	Sequential	Heuristic	Integrated O.	Integrated F.
Unsolved Instances	22	2	21	14
Solved Instances	59	79	60	67
Solved Optimally	9	9	9	9
Best Solution	3	40	11	35
Shortest Runtime	7	11	9	0

Table 1: Results within 6 hours minimizing costs

Regarding the freshness objective we tested the integrated approach and the heuristic and present the results in Table 2. Since the optimal solution is not known, the solution quality of the heuristic could not be evaluated but the gap could be determined.

	Integrated Approach	Heuristic
Unsolved Instances	47	33
Solved Instances	34	48
Solved Optimally	0	
Average Gap	76%	92%

Table 2: Results within 6 hours optimizing freshness

## 5 Pareto-Optimal Front

The points of the Pareto-optimal fronts were determined with a maximal computation time of three hours per optimization run. In Figure 1 we present two Pareto-optimal fronts for small problem instances.

The run of the curves shows that, starting at the point with the lowest cost, the freshness could be increased with just a bit higher cost. Further improvements regarding freshness are only possible by accepting significantly higher costs.

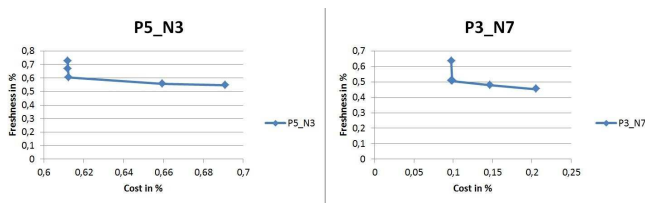


Figure 1: Pareto-optimal fronts of two instances

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## List of Participants

**Absi, Nabil**

Ecole des Mines de Saint-Etienne  
absi@emse.fr

**Aggoune, Riad**

Tudor  
Riad.Aggoune@tudor.lu

**Agra, Agostinho**

University of Aveiro  
aagra@ua.pt

**Akartunali, Kerem**

University of Strathclyde  
kerem.arkartunali@strath.ac.uk

**Akbalik, Ayse**

Université de Lorraine  
ayse.akbalik@univ-metz.fr

**Almada-Lobo, Bernardo**

Porto University  
almada.lobo@fe.up.pt

**Almeder, Christian**

European University Viadrina Frank-  
furt(Oder)  
Almeder@europa-uni.de

**Amorim, Pedro**

Porto University  
pamorim@fe.up.pt

**Burmeister, Kristina**

Leibniz University Hannover  
burmeister@prod.uni-hannover.de

**Carvalho, Margarida**

Porto University  
margarida.carvalho@dcc.fc.up.pt

**Cherri, Luiz**

University of Sao Paulo  
luizcherri@gmail.com

**Copil, Karina**

University of Cologne  
copil@wiso.uni-koeln.de

**Curcio, Eduardo**

Porto University  
efcurcio@hotmail.com

**Dauzere-Peres, Stephane**

Ecole Nationale Supérieure des  
Mines de Saint-Etienne  
Dauzere-Peres@emse.fr

**Delgado, Alexandrino**

University of Aveiro  
alexandrino@ua.pt

**Doostmohammadi, Mahdi**

University of Strathclyde  
mahdi.doostmohammadi@strath.ac.uk

**Düerkop, Sascha**

University of Cologne  
dueerkop@wiso.uni-koeln.de

**Elias, Andreas**

University of Duisburg-Essen  
andreas.elias@uni-due.de

**Figueira, Luis Gonçalo**

Porto University  
luis.figueira@fe.up.pt

**Fragkos, Ioannis**

University College London  
i.fragkos@ucl.ac.uk

**Gicquel, Celine**

Universite Paris Sud  
Celine.Gicquel@lri.fr

**Gomez Urrutia, Edwin**

Tudor  
Edwin.Gomez@tudor.lu

**Goren, Hacer**

Pamukkale University  
hgoren@pau.edu.tr

**Guimarães, Luís**

Porto University  
guimaraes.luis@fe.up.pt

**Helber, Stefan**

Leibniz University Hannover  
stefan.helber@prod.uni-hannover.de

**Herrmann, Frank**

OTH Regensburg  
Frank.Herrmann@OTH-Regensburg.de

**Hilger, Timo**

University of Cologne  
timo.hilger@wiso.uni-koeln.de

**Jans, Raf**

HEC Montréal  
raf.jans@hec.ca

**Kedad-Sidhoum, Safia**

Laboratory of Computer Sciences,  
Paris 6 (UPMC)  
safia.kedad-sidhoum@lip6.fr

**Kian, Ramez**

Bilkent University  
ramezk@bilkent.edu.tr

**Kiliç, Onur**

Hacettepe University  
onuralp@hacettepe.edu.tr

**Kimms, Alf**

University of Duisburg-Essen  
alf.kimms@uni-due.de

**Martins, Sara**

Porto University  
sara.martins@fe.up.pt

**Meyr, Herbert**

Hohenheim University  
H.Meyr@uni-hohenheim.de

**Michelli, Maldonado**

Universidade Estadual Paulista-  
UNESP  
michellimaldo@gmail.com

**Moreira, Fábio**

Porto University  
fnssm26@gmail.com

**Pedroso, João P.**

Porto University  
jpp@fc.up.pt

**Rapine, Christophe**

University of Lorraine  
christophe.rapine@univ-lorraine.fr

**Rowshannahad, Mehdi**

Ecole des Mines de Saint-Etienne  
rowshannahad@gmail.com

**Sahling, Florian**

Leibniz University Hannover  
florian.sahling@prod.uni-hannover.de

**Santos, Maristela Oliveira dos**

University of Sao Paulo  
mari@icmc.usp.br

**Schimmelpfeng, Katja**

University of Hohenheim  
Katja.Schimmelpfeng@uni-hohenheim.de

**Solyali, Oguz**

Middle East Technical University  
solyali@metu.edu.tr

**Sterenzky, Christophe**

Ecole des Mines de Saint-Etienne  
christophe.sterenzky@gmail.com

**Sural, Haldun**

Middle East Technical University  
hsural@metu.edu.tr

**Tempelmeier, Horst**

University of Cologne  
tempelmeier@wiso.uni-koeln.de

**Tunc, Huseyin**  
Hacettepe University  
huseyin.tunc@hacettepe.edu.tr

**van den Heuvel, Wilco**  
Erasmus University Rotterdam  
wvandenheuvel@ese.eur.nl

**Van Vyve, Mathieu**  
Université catholique de Louvain  
mathieu.vanvyve@uclouvain.be

**Vogel, Tom**  
Europa University  
tvogel@europa-uni.de

**Wagelmans, Albert P.M.**  
Erasmus University Rotterdam  
wagelmans@few.eur.nl

**Wei, Wenchao**  
Université catholique de Louvain  
Wenchao.Wei@kuleuven.be

**Woerbelauer, Martin**  
Universität Hohenheim  
M.Woerbelauer@uni-hohenheim.de

**Wolsey, Laurence A.**  
Université catholique de Louvain  
laurence.wolsey@uclouvain.be